The HBS effect with extensive margins[1]

[Very Preliminary]
Comment welcome

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Abstract

This paper analyses the Harrod-Barassa-Samuelson (HBS) effect (and its components such as the international relative wage and the terms of trade) with extensive margins. The variation of the number of available varieties has an important consequence on the determination of the international relative prices and international transmission problem. Especially we emphasize the role played by the elasticity of substitution between traded and nontraded consumption bundle. Except the Cobb-Douglas case, a non unity elasticity creates the variation of relative spending and, as a consequence, that of relative profits between traded and nontraded sector which induces the entry and exit of variety representing firms.

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1 Introduction

The HBS effect, discussed often as a real purchasing power increase of one country, provides a lot of rich problematic in open macroeconomics today. Synthetically this effect is configurated as follows: A country which hit by a positive technology shock at its traded sector experiences a variation of its terms of trade (TOT) and at the same time the relative wage appreciation which, given the perfect mobility of labor allocation between two sectors, also induces an appreciation of their nontradables price. The sign of the real exchange rate (RER) is determined by the relative importance of TOT movement and the relative price of nontradables.

Recently Corsetti, Dedola and Luduc (2006) reports a mixed result of their VAR estimation among G7 countries. They report TOT and RER appreciation for US and Japan, their depreciation for UK and Italy, and Germany as a middle case of these two groups. Broadly there are two types of categories of model which explain this mixed TOT movement. One supposes only "intensive margins" (the variation of the quantities of a fixed number of varieties) and another permits also the "extensive margins" (the variation of the number of varieties). At the first type of model a positive technology shock at tradable sector induces more intensive margins which are absorbed at a lower price in international market (TOT depreciation). As a consequence a positive international transmission takes place via lower imported price to foreigners. At the second type of model, a possible adjustment by the extensive margins after a technological improvement would reverse the TOT movement. A wider range of tradable varieties which attract the demand worldwide would result in this country's higher tradables price (TOT appreciation). The foreigners would be hearted by an more expensive imported goods price but they have a gains in terms of a wider range of available imported varieties.

The role played by extensive margins has become emphasized in the related literature. Broda and Weinstein (2004) measure the welfare gains which comes from the extensive margins and point out the upward bias of CPI. Based on the heterogenous firms' structure in terms of productivity, Ghironi and Melitz (2005) argue the TOT appreciation by extensive margins and its consequence on the HBS effect. Also based on the heterogenous firms Bergin, Glick and Taylor (2006) emphasizes the endogenous improvement of the traded sector's productivity which is a "precondition" for the HBS effect. Corsetti, Martin and Pesenti (henceforth CMP (2006)) analyses the TOT appreciation by extensive margins and its transmission problem in their static model. Also in their second paper CMP (2007), they argue the impact of the US current account adjustment including the extensive margins and point out a possibly smaller needed TOT deterioration for US.

Model's feature and intuition

Our paper argue the possibility of TOT appreciation from the extensive margins and its consequence on international relative prices. Theoretical framework is a direct expansion of CMP (2006) where we introduce the nontradable sector. As them our analysis focuses on the long term relationship imposing a balanced trade condition.[3] However contrasting to them, we emphasize the interaction between tradable and nontradable sector, especially the role played by the elasticity of substitution. Except the case of Cobb-Douglas consumption bundle, this creates the variation of relative spending on two sectors, hence that of profits inducing the entry and exist in each sector. This effect comes endogenously from "the demand side" of the economy contrasting to the supply side productivity shock such as an exogenous improvement of firms' setting up technology which induce an positive extensive margins on impact. Together with relative degree of such supply side shock, this demand side effect creates an additional variation of the extensive margins from sector to sector and one country to another. Nothing to say about the additional gains or lose in international transmission in terms of extensive margins, this affects also the degree of the movement of international relative prices such as relative wage, terms of trade and real exchange rate.

The structure of the paper is as follows. First we present our model in the next section. The equilibrium conditions are given at the end. In section 3, we discuss the implication of the model at symmetric steady state equilibrium. Here especially the steady state ratio between Home traded goods and non traded is analyzed in detail. In section 4 the analytical result for Cobb-Douglas consumption bundle between traded and nontraded goods is presented. In the next section we report the implication of other values of the elasticity of substitution on the variation of the number of varieties and on the international relative prices with a numerical example. At the end a brief conclusion will be given.

2 The model

We construct a model of monopolistic competition where there are two countries, Home and Foreign. Foreign variables are denoted with a star. In each country there are unit mass of population and two types of firms : one is that participates in exporting activity $h_T \in [0, n_T]$ ($f_T \in [0, n_T^*]$) and another is specialized only on domestic sales $h_N \in [0, n_N]$ ($f_N \in [0, n_N^*]$). Each firm in each sector represents one variety and its number is determined endogenously. We abstract from heterogeneity of firms in productivity (production level or entry cost level) supposing at a long term equilibrium relationship where the productivity level has converged. Governments' expenditures are also excluded for simplicity.

2-1 Households

As in CMP (2006) we choose $w = 1$ as numeraire for the simplicity of analysis. The home representative household's utility is defined as a positive

[3] For simplicity labor supply is inelastic and the love for variety is Dixt-Stigliz type in our model.

function of aggregated consumption level C and a negative function of labor supply l :

$$U = \ln C - l \tag{1}$$

The consumption basket C is composed by C_T (consumption in traded sector) and C_N (consumption in non-traded sector).

$$C = \left[\delta^{\frac{1}{\rho}} C_T^{1-\frac{1}{\rho}} + (1-\delta)^{\frac{1}{\rho}} C_N^{1-\frac{1}{\rho}} \right]^{\frac{1}{1-\frac{1}{\rho}}} \tag{2}$$

where δ $(1-\delta)$ is the preference weight on traded (non traded) sector. And ρ is the elasticity of substitution between two sector's consumption basket C_T and C_N. We impose from the start the "love of variety" of consumers with Dixit-Stigliz type of preference. (So the marginal utility of consuming one additional variety is $1/\sigma - 1.$[4])

$$C_T = \left[\int_0^{n_T} c(h_T)^{1-\frac{1}{\sigma}} \, dh_T + \int_0^{n_T^*} c(f_T)^{1-\frac{1}{\sigma}} \, df_T \right]^{\frac{1}{1-\frac{1}{\sigma}}} \tag{3}$$

$$C_N = \left[\int_0^{n_N} c(h_N)^{1-\frac{1}{\sigma}} \, dh_N \right]^{\frac{1}{1-\frac{1}{\sigma}}} \tag{4}$$

where n_T (n_T^*) is the number of available varieties in Home traded goods (Foreign traded goods). n_N is that of Home non-traded goods. $c(h_T)$, $c(f_T)$ and $c(h_N)$ are the demand for each domestic tradable, foreign tradable and domestic non traded individual variety for the representative household. σ is the elasticity of substitution among varieties. As we will discuss later, "Home bias" in traded sector's consumption is captured in the form of transportation cost (ϕ). We assume conventionally $\sigma > 1$, $\rho > 0$ and $0 \leq \delta \leq 1$.

The Price index (welfare-based) which corresponds to this consumption structure is :

$$P = \left[\delta P_T^{1-\rho} + (1-\delta) P_N^{1-\rho} \right]^{\frac{1}{1-\rho}} \tag{5}$$

and the price indices for each sector :

$$P_T = \left[\int_0^{n_T} p(h_T)^{1-\sigma} \, dh_T + \int_0^{n_T^*} p(f_T)^{1-\sigma} \, df_T \right]^{\frac{1}{1-\sigma}} \tag{6}$$

[4] As it is treated in CMP (2006) the degree of "love of variety" is not necessarily connected to σ. It would be represented as a function of an another exogenous parameter. See Benassy (1996)

$$P_N = \left[\int_0^{n_N} p(h_N)^{1-\sigma} \, dh_N \right]^{\frac{1}{1-\sigma}} \tag{7}$$

where $p(h_T)$, $p(f_T)$ and $p(h_N)$ are the price of individual variety in the Home. The representative household finances equally entirely fixed cost needed for each domestic firm's setting up, $q(h_T)$ and $q(h_N)$ and receive equally its profits, $\pi(h_T)$ and $\pi(h_N)$:

$$I \equiv \int_0^{n_T} q(h_T) \, dh_T + \int_0^{n_N} q(h_N) \, dh_N \tag{8}$$

$$\Pi \equiv \int_0^{n_T} \pi(h_T) \, dh_T + \int_0^{n_N} \pi(h_N) \, dh_N \tag{9}$$

where I and Π are total investments and received dividend in the Home economy. Finally we can write the budget constraint for representative domestic household as follows :

$$\int_0^{n_T} p(h_T) \, c(h_T) \, dh_T + \int_0^{n_T^*} p(f_T) \, c(f_T) \, df + \int_0^{n_N} p(h_N) \, c(h_N) \, dh_N + I \leq l + \Pi \tag{10}$$

Households maximize their utilities under above constraint. Optimal aggregate consumption and labor supply are :

$$C = P^{-1}, \qquad l = 1 \tag{11}$$

Optimal consumptions for each sector are given by :

$$C_T = \delta \left(\frac{P_T}{P} \right)^{-\rho} C, \qquad C_N = (1-\delta) \left(\frac{P_N}{P} \right)^{-\rho} C \tag{12}$$

Optimal consumptions for each individual variety are:

$$c(h_T) = \left(\frac{p(h_T)}{P_T} \right)^{-\sigma} C_T, \qquad c(f_T) = \left(\frac{p(f_T)}{P_T} \right)^{-\sigma} C_T \tag{13}$$

$$c(h_N) = c(h_T) = \left(\frac{p(h_N)}{P_N} \right)^{-\sigma} C_N \tag{14}$$

There is no investment choice in this economy. Similar expressions hold for the Foreign.

2-2 Firms

Individual firms in each sector in each country have sector-country-specifique liner technology in labor to produce final goods :

$$Y\left(h_T\right)=\alpha_T l(h_T), \qquad Y\left(h_N\right)=\alpha_N l(h_N) \tag{15}$$

where $Y(h_T)$ and $Y\left(h_N\right)$ is the output of individual variety in each sector and α_T and α_N are labor productivity for production. $l(h_T)$ and $l(h_N)$ are labor used in production. To start the production firms in each sector need $1/v_T$ and $1/v_N$ units of labor respectively. So the starting up fixed cost for each individual variety is :

$$q\left(h_T\right)=1/v_T, \qquad q\left(h_N\right)=1/v_N \tag{16}$$

Home tradable varieties are exported to the Foreign with "iceberg" type transportation cost, τ. The resource constraints for each sector's individual firm become :

$$Y\left(h_T\right)\geq c(h_T)+(1+\tau)c^*(h_T) \tag{17}$$

$$Y\left(h_N\right)\geq c(h_N) \tag{18}$$

and the profit of individual firm in each tradable and non traded sector is as follows :

$$\pi\left(h_T\right)\equiv p(h_T)c(h_T)+\varepsilon p^*(h_T)c^*(h_T)-l(h_T) \tag{19}$$

$$\pi\left(h_N\right)\equiv p(h_N)c(h_N)-l(h_N) \tag{20}$$

for tradable and non traded firm respectively. ε is the exchange rate defined as foreign real wage in terms of home labor units. Individual optimal variety price is determined by firm's monopolistic behavior as follows :

$$p\left(h_T\right)=\frac{\sigma}{\sigma-1}\frac{1}{\alpha_T}\equiv p_T \tag{21}$$

Home variety price in Foreign country once converted in Home currency must be equal to $p\left(h_T\right)$ without trade friction,τ in the long run flexible price :

$$\varepsilon p^*\left(h_T\right)=\frac{\sigma}{\sigma-1}\frac{1}{\alpha_T}\left(1+\tau\right)=p_T\left(1+\tau\right) \tag{22}$$

In the same token the individual price of home non-traded variety:

$$p\left(h_N\right)=\frac{\sigma}{\sigma-1}\frac{1}{\alpha_N}\equiv p_N \tag{23}$$

With a symmetry at equilibrium we can write,

$$P_T=p_T\Omega^{\frac{1}{1-\sigma}}, \qquad P_N=p_N n_N^{\frac{1}{1-\sigma}} \tag{24}$$

where

$$\Omega \equiv n_T + n_T^* \phi \left(\varepsilon p_T^*/p_T\right)^{1-\sigma}, \qquad \Omega^* \equiv n_T^* + n_T \phi \left(\varepsilon p_T^*/p_T\right)^{\sigma-1} \quad (25)$$

$$\phi \equiv (1+\tau)^{1-\sigma} \quad (26)$$

in which τ is a transportation cost. So $0 \leq \phi \leq 1$ and $\phi \to 1$ means trade liberalization.

2-3 Profits and long term equilibrium conditions

The profits for each sector are given by :

$$\pi(h_T) = \frac{\delta p_T^{1-\rho}}{\sigma} \left[\frac{1}{\Omega^{\frac{\rho-\sigma}{1-\sigma}} P^{1-\rho}} + \phi \frac{\varepsilon^{\sigma} \left(p_T^*/p_T\right)^{\sigma-\rho}}{\Omega^{*\frac{\rho-\sigma}{1-\sigma}} P^{*1-\rho}} \right] \equiv \pi_T \quad (27)$$

$$\pi(h_N) = \frac{(1-\delta)\, p_N^{1-\rho}}{\sigma} \frac{1}{n_N^{\frac{\rho-\sigma}{1-\sigma}} P^{1-\rho}} \equiv \pi_N \quad (28)$$

π_T and π_N are the profits using symmetry among the firms. Similar expressions hold for the Foreign. In the long term equilibrium, we have 4 free entry conditions:

$$\pi_T = \frac{1}{v_T}, \qquad \pi_N = \frac{1}{v_N} \quad (29)$$

$$\pi_T^* = \frac{1}{v_T^*}, \qquad \pi_N^* = \frac{1}{v_N^*} \quad (30)$$

where v_T, v_T^*, v_N and v_N^* are respectively firm setting up (innovation) productivity in Home (Foreign) traded (non-traded) sector. Note at equilibrium quantities of the production of each firm (intensive margins) is given by:

$$Y(h_T) = (\sigma-1)\frac{\alpha_T}{v_T}, \qquad Y(h_N) = (\sigma-1)\frac{\alpha_N}{v_N} \quad (31)$$

With a marginal cost shock, the intensive margins appears but that decreases with a entry cost shock. The analogous expressions hold for foreign firms. Finally because we suppose a financial autarky, the trade must be balanced :

$$\frac{\delta\phi}{\sigma} \left[\frac{n_T p_T^{1-\rho} \varepsilon^{\sigma} \left(p_T^*/p_T\right)^{\sigma-\rho}}{\Omega^{*\frac{\rho-\sigma}{1-\sigma}} P^{*1-\rho}} - \frac{n_T^* p_T^{*1-\rho} \varepsilon^{1-\sigma} \left(p_T^*/p_T\right)^{\rho-\sigma}}{\Omega^{\frac{\rho-\sigma}{1-\sigma}} P^{1-\rho}} \right] = 0 \quad (32)$$

Behind of these equilibrium conditions, the labor demand and supply in the production and the creation of varieties in both sector must be equal. Using above expressions the total labor demand becomes :

$$\int_0^{n_T} \frac{Y(h_T)}{\alpha_T} dh_T + \int_0^{n_N} \frac{Y(h_N)}{\alpha_N} dh_N + \int_0^{n_T} \frac{1}{v_T} dh_T + \int_0^{n_N} \frac{1}{v_N} dh_N = \sigma(\frac{n_T}{v_T} + \frac{n_N}{v_N}) \quad (33)$$

at equilibrium,

$$\sigma(\frac{n_T}{v_T} + \frac{n_N}{v_N}) = 1 \quad (34)$$

Similar labor market clear condition holds for the Foreign.

At the end we have 7 equilibrium conditions (4 free entry conditions, 2 labor market clearing conditions and one trade balance condition) and 5 endogenous variables : $n_{T,}$ $n_{N,}$ n_T^*, n_N^* and ε. For the solution of the model two labor market clearing conditions are redundant. Because of our choice of numeraire all nominal variables are expressed in terms of home labor units. Any movement of the ε from its steady state value captures the relative real wage movement between Home and Foreign. A higher ε means a depreciation of home labor value.

Using above trade balance conditions and profits (28) (29), we can write the profits in each sector as :

$$\pi_T = \frac{P_T C_T}{\sigma n_T}, \qquad \pi_N = \frac{P_N C_N}{\sigma n_N} \quad (35)$$

The analogous expressions hold for foreign firms. These expressions highlight how the number of varieties affect the profits. For each sector's individual firms its profits is decreasing with a expansion of the number of varieties and increasing with spending on that sector. We can calculate a sufficient condition on parameters which permits that the number of varieties doesn't increase infinitely after a positive shock on entry cost. That is:

$$\frac{(\rho - 1)(1 - \delta)}{\sigma - 1} < 1 \quad (36)$$

Obstfeld and Rogoff (2005) argue the value of ρ is between 0.5 and 2. σ takes a relatively large value compared to ρ. Then supposing $\rho \leq \sigma$ and $\sigma > 1$ is not so restricting. These ordering of parameters verify the above condition, hence after the productivity shock the number of varieties remains in a finite number.

3 Symmetric steady state

Our model's parameters are σ, ρ, δ and ϕ. At the symmetric steady state, we set $\overline{\alpha_T} = \overline{\alpha_N} = \overline{\alpha_T^*} = \overline{\alpha_N^*} = \overline{v_T} = \overline{v_N} = \overline{v_T^*} = \overline{v_N^*} = 1$. It is straightforward at such symmetric equilibrium, $\overline{n_T} = \overline{n_T^*}$, $\overline{n_N} = \overline{n_N^*}$. And from the trade balance condition we get $\overline{\varepsilon} = 1$. (may be in Appendix) For the explicit steady state solution of $\overline{n_T}$ and $\overline{n_N}$, using the demand functions (12) and $PC = 1$, we can write the profits in two sector as:

$$\pi_T = \frac{1}{\sigma n_T}\delta\left(\frac{P_T}{P}\right)^{1-\rho}, \qquad \pi_N = \frac{1}{\sigma n_N}(1-\delta)\left(\frac{P_N}{P}\right)^{1-\rho} \tag{37}$$

From the definition of the price indices, P_T and P_N the steady state relative price is given by:

$$\frac{\overline{P_T}}{\overline{P_N}} = \left(\frac{\overline{n_N}}{\overline{n_T}}\frac{1}{1+\phi}\right)^{\frac{1}{\sigma-1}} \tag{38}$$

Noting $\overline{\pi_T} = \overline{\pi_N}$, and taking the steady state ratio of profits between two sectors from (37) then using (38) we get the steady state ratio of the number of varieties which we define as λ :

$$\lambda \equiv \frac{\overline{n_T}}{\overline{n_N}} = \left(\frac{\delta}{1-\delta}\right)^{\frac{\sigma-1}{\sigma-\rho}}(1+\phi)^{\frac{\rho-1}{\sigma-\rho}} \tag{39}$$

Under the parametric restriction we have seen, a larger preference weight on traded goods (an increase of δ) increases the number of home tradeable. Theoretically λ can change from 0 to infinity in function of δ. Here an increase of ϕ (trade liberalization) doesn't always mean a corresponding expansion of the number of traded varieties. It depends crucially on the value of ρ. When $\rho > 1$, with an exogenous trade liberalization($\phi \to 1$), the number of traded varieties increases relative to that of non-traded sector. When $0 < \rho < 1$, the effect is inverse. In short this comes from the fact that with a trade liberalization which decreases the Home traded price index P_T, the Home traded sector's firms don't have an enough increased demand of their goods (which is in C_T) to get an increased spending compared to nontraded goods ($P_T C_T / P_N C_N$) under a low elasticity of substitution. This means the relatively lower profits of traded sector and lower number of varieties.[5]

In the case of $\rho = 1$, the steady state relative number of varieties is given by :

$$\lambda = \frac{\delta}{1-\delta} \tag{40}$$

which depends only on relative spending weight δ.

4 Analytical result for different productivity shocks under $\rho = 1$

In what follows, we analyze the effect of 4 different types of asymmetric positive productivity shock ($d\alpha_T$, dv_T, $d\alpha_N$ and dv_N) on the number of varieties in each sector of both country (n_T, n_N, n_T^* and n_N^*), on the relative wage (ε).

[5] With a symmetric trade liberalization Home traded sector's firms earn more at Foreign market and lose more at domestic market (because of increased competition with Foreign firms). But because of the balanced trade condition, we can write traded sector's firms' profit as a function of domestic spending on that sector, $P_T C_T$. This fact has a crucial meaning for the determination of the number of varieties. What matter is the domestic relative spending between traded and nontraded goods. This point will be discussed at section 5.

To get the intuition about the general mechanism of the model we set $\rho = 1$ (Cobb-Douglas consumption bundle between traded and nontraded basket) at first and present its analytical result. We define the terms of trade (TOT) and the real exchange rate (RER) as :

$$TOT \equiv \frac{\varepsilon p_T^*}{p_T}, \qquad RER \equiv \frac{\varepsilon P^*}{P} \tag{41}$$

So the increase of TOT and RER mean the real depreciation for Home. In the spirt of Ghironi and Melitz (2005), we also give the results for empirical counterpart of each index in which the variation of the number of varieties is abstracted. Such indices are noted with $\tilde{}$.

The analytical result about the shocks on home traded sector is shown below:

	$d\alpha_T > 0$	$dv_T > 0$
$\frac{1}{\overline{n_T}}\frac{dn_T}{d\alpha_T}$	0	1
$\frac{1}{\overline{n_N}}\frac{dn_N}{d\alpha_T}$	0	0
$\frac{1}{\overline{n_T}}\frac{dn_T^*}{d\alpha_T}$	0	0
$\frac{1}{\overline{n_N}}\frac{dn_N^*}{d\alpha_T}$	0	0
$\frac{1}{\overline{\varepsilon}}\frac{d\varepsilon}{d\alpha_T}$	$-\frac{2(\sigma-1)}{2\sigma-1+\phi} < 0$	$-\frac{2}{2\sigma-1+\phi} < 0$
$\frac{1}{\overline{TOT}}\frac{dTOT}{d\alpha_T}$	$\frac{1+\phi}{2\sigma-1+\phi} > 0$	$-\frac{2}{2\sigma-1+\phi} < 0$
$\frac{1}{\overline{RER}}\frac{dRER}{d\alpha_T}$	$\frac{(1-\phi)\delta-2(\sigma-1)(1-\delta)}{2\sigma-1+\phi}$	$\frac{(1-\phi)\delta-2(\sigma-1)(1-\delta)}{(\sigma-1)(2\sigma-1+\phi)}$
$\frac{1}{\overline{C}}\frac{dC}{d\alpha_T}$	$\frac{\delta(2\sigma-1)}{2\sigma-1+\phi} > 0$	$\frac{\delta(2\sigma-1)}{(\sigma-1)(2\sigma-1+\phi)} > 0$
$\frac{1}{\overline{C}}\frac{dC^*}{d\alpha_T}$	$\frac{\delta\phi}{2\sigma-1+\phi} > 0$	$\frac{\delta\phi}{(\sigma-1)(2\sigma-1+\phi)} > 0$
$\frac{1}{\overline{\widetilde{RER}}}\frac{d\widetilde{RER}}{d\alpha_T}$	$\frac{(1-\phi)\delta-2(\sigma-1)(1-\delta)}{2\sigma-1+\phi}$	$-\frac{2}{2\sigma-1+\phi}\left[\frac{1-\phi}{1+\phi}\delta+1-\delta\right] < 0$
$\frac{1}{\overline{C}}\frac{d\widetilde{C}}{d\alpha_T}$	$\frac{\delta(2\sigma-1)}{2\sigma-1+\phi} > 0$	$\frac{2\delta\phi}{(1+\phi)(2\sigma-1+\phi)} > 0$
$\frac{1}{\overline{C}}\frac{d\widetilde{C^*}}{d\alpha_T}$	$\frac{\delta\phi}{2\sigma-1+\phi} > 0$	$-\frac{2\delta\phi}{(1+\phi)(2\sigma-1+\phi)} < 0$

In case of a positive marginal cost shock ($d\alpha_T > 0$) there is no change of the number of varieties both in Home and Foreign. Home countries' intensive margins are absorbed with a lower international price (TOT depreciates) in spite of the Home real cost appreciation (a negative variation of ε). As a consequence a positive transmission takes place to Foreign who enjoys lower imported goods' price. The response of the RER is determined by a relative importance of TOT movement and relative nontraded goods price across countries. In this case the sign of variation is ambiguous but tend to appreciate with a higher spending weight on nontraded basket $(1-\delta)$. Because there is no extensive margins, the welfare-consistent indices coincide with that of empirical based.[6] [7]

When an entry cost shock $(dv_T > 0)$ takes place, 1% increase of the number of varieties at Home tradable sector appears. These extensive margins, under Dixit-Stiglitz type of love for variety, are demanded immediately in world market appreciating the international relative price for Home traded goods (Home relative wage and TOT appreciates). A negative transmission takes place with a higher imported goods' price to Foreign, however its sign is reversed if we take into account a positive transmission in terms of a wider range of available varieties. The sign of the welfare based RER is ambiguous but that of empirical based now shows a clear appreciation reflecting the TOT appreciation. Note as in the case of marginal cost shock, this RER appreciation becomes stronger with a higher spending weight on nontradables.

Observe also the international transmission happens more "smoothly" with a higher ϕ (a lower iceberg type transportation cost) both in terms of intensive and extensive margins.

Next we see the analytical result about the shocks on home nontraded sector which are given by the table below:

[6] Under $\rho = 1$ with a marginal cost shock, no variety effect takes place and TOT depreciates always with $\sigma > 1$. There is no TOT reversal associated with a higher negative income effect on home traded goods for home agents under low imported elasticity as discussed in Corsetti, Dedola and Luduc (2006)

[7] With a marginal cost shock on traded sector, setting $\rho = 1$ and zero transportation cost it is easy to find "the simplest text book HBS effect" where there is no TOT movement. See Appendix 3

	$d\alpha_N > 0$	$dv_N > 0$
$\frac{1}{n_T}\frac{dn_T}{d\alpha_T}$	0	0
$\frac{1}{n_N}\frac{dn_N}{d\alpha_T}$	0	1
$\frac{1}{n_T}\frac{dn_T^*}{d\alpha_T}$	0	0
$\frac{1}{n_N}\frac{dn_N^*}{d\alpha_T}$	0	0
$\frac{1}{\varepsilon}\frac{d\varepsilon}{d\alpha_T}$	0	0
$\frac{1}{TOT}\frac{dTOT}{d\alpha_T}$	0	0
$\frac{1}{RER}\frac{dRER}{d\alpha_T}$	$1-\delta > 0$	$\frac{1-\delta}{\sigma-1} > 0$
$\frac{1}{C}\frac{dC}{d\alpha_T}$	$1-\delta > 0$	$\frac{1-\delta}{\sigma-1} > 0$
$\frac{1}{C}\frac{dC^*}{d\alpha_T}$	0	0
$\frac{1}{\widetilde{RER}}\frac{d\widetilde{RER}}{d\alpha_T}$	$1-\delta > 0$	0
$\frac{1}{\widetilde{C}}\frac{d\widetilde{C}}{d\alpha_T}$	$1-\delta > 0$	0
$\frac{1}{\widetilde{C}}\frac{d\widetilde{C}^*}{d\alpha_T}$	0	0

As in the case of traded sector with a marginal cost shock ($d\alpha_N > 0$) there is no change of the number of varieties. Under $\rho = 1$, no movement on international relative wage and TOT. The intensive margins at nontraded sector are absorbed at Home with a lower price. Only Home enjoys a higher level of consumption reflecting this technological improvement. There is no transmission abroad. Home's real purchasing power decreases reflecting this cheaper nontradables compared to Foreign (RER and $\widetilde{RER}$ depreciate). As expected this RER depreciation becomes stronger with a higher spending on nontradables.

When an entry cost shock hits at nontraded sector ($dv_N > 0$), there is no variation on relative wage and TOT, however the extensive margins appears at home nontraded sector. This improves Home welfare-consistent consumption and depreciates the welfare based RER. After eliminating these gains in terms of extensive margins, nothing happens. We see no variation on empirical based indices.

5 The role of the elasticity of substitution between tradables and nontradables on the determination of the number of varieties and its consequence on the international relative prices

As we have seen, under $\rho = 1$ there is no variation of the number of varieties except in the case of entry cost shock on the sector concerned. This is no more the case when $\rho \neq 1$ which creates a rich demand side effect on the variation of the number of varieties. And this variation of extensive margins provides an important consequence on international relative prices such as TOT and RER. To see this, we present an analytical explanation at first and next we comment on a numerically simulated results under different value of ρ.

Using (35) and free entry conditions (29), for any type of shock dx the total differentiation of the relative profits between traded and nontraded sector at Home gives:

$$\frac{1}{\overline{n_T}}\frac{dn_T}{dx}-\frac{1}{\overline{n_N}}\frac{dn_N}{dx}=\left[\left(\frac{1}{\overline{\overline{P_T}}}\frac{dP_T}{dx}-\frac{1}{\overline{\overline{P_N}}}\frac{dP_N}{dx}\right)+\left(\frac{1}{\overline{\overline{C_T}}}\frac{dC_T}{dx}-\frac{1}{\overline{\overline{C_N}}}\frac{dC_N}{dx}\right)\right]+\left(\frac{dv_T}{dx}-\frac{dv_N}{dx}\right) \tag{42}$$

which says that the variation of the relative number between two sector proportionally depends on the relative expenditure for welfare consistent baskets (in terms of home labor units because of our choice of numeraire) and relative strength of entry cost shock. The right hand first big parentheses is the demand side effect inducing the change of the left hand side of the free entry conditions (29). The last term at right hand of the equation presents the supply side effect inducing the change of the right hand side of the free entry conditions directly.

To get more closely into the problem we find it is nice to separate the substitution and income effect using the Slutsky equation on the variation of the demand for both traded and nontraded basket. For the consumption basket of Home including traded goods, C_T :

$$\frac{1}{\overline{\overline{C_T}}}\frac{dC_T}{dx}=\left\{\begin{array}{c}\underbrace{-\rho\left(\frac{1}{\lambda+1}\right)\left(\frac{1}{\overline{\overline{P_T}}}\frac{dP_T}{dx}-\frac{1}{\overline{\overline{P_N}}}\frac{dP_N}{dx}\right)}_{Substitution\ Effect}\\ \underbrace{-\frac{1}{\overline{\overline{P_T}}}\frac{dP_T}{dx}+\left(\frac{1}{\lambda+1}\right)\left(\frac{1}{\overline{\overline{P_T}}}\frac{dP_T}{dx}-\frac{1}{\overline{\overline{P_N}}}\frac{dP_N}{dx}\right)}_{Income\ Effect}\end{array}\right\} \tag{43}$$

For Home nontraded basket, C_N :

$$\frac{1}{\overline{\overline{C_N}}}\frac{dC_N}{dx}=\left\{\begin{array}{c}\underbrace{-\rho\left(\frac{1}{\lambda+1}\right)\left(\frac{1}{\overline{\overline{P_N}}}\frac{dP_N}{dx}-\frac{1}{\overline{\overline{P_T}}}\frac{dP_T}{dx}\right)}_{Substitution\ Effect}\\ \underbrace{-\frac{1}{\overline{\overline{P_N}}}\frac{dP_N}{dx}+\left(\frac{1}{\lambda+1}\right)\left(\frac{1}{\overline{\overline{P_N}}}\frac{dP_N}{dx}-\frac{1}{\overline{\overline{P_T}}}\frac{dP_T}{dx}\right)}_{Income\ Effect}\end{array}\right\} \tag{44}$$

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In relative term:

$$\frac{1}{\overline{C_T}}\frac{dC_T}{dx}-\frac{1}{\overline{C_N}}\frac{dC_N}{dx}=\left\{\begin{array}{c}\underbrace{-\rho\left(\frac{2}{\lambda+1}\right)\left(\frac{1}{\overline{P_T}}\frac{dP_T}{dx}-\frac{1}{\overline{P_N}}\frac{dP_N}{dx}\right)}_{Substitution\ Effect}\\ \underbrace{-\left(\frac{\lambda}{\lambda+1}-\frac{1}{\lambda+1}\right)\left(\frac{1}{\overline{P_T}}\frac{dP_T}{dx}-\frac{1}{\overline{P_N}}\frac{dP_N}{dx}\right)}_{Income\ Effect}\end{array}\right\} \tag{47}$$

As we can see the coefficient on the substitution effect is a weighted ρ by the relative importance of traded and nontraded varieties, λ. The sign of the income effect is determined by λ. For example a higher preference weight on nontraded goods ($\delta < 1/2$) makes λ go to 0, leading $-(\lambda/\lambda+1-1/\lambda+1)$ to 1. Strong negative income effect works on the relative consumption when the traded basket becomes cheaper in such case. Putting this relationship in (42) and arranging the terms finally we get:

$$\frac{1}{\overline{n_T}}\frac{dn_T}{dx}-\frac{1}{\overline{n_N}}\frac{dn_N}{dx}=-(\rho-1)\left(\frac{2}{\lambda+1}\right)\left(\frac{1}{\overline{P_T}}\frac{dP_T}{dx}-\frac{1}{\overline{P_N}}\frac{dP_N}{dx}\right)+\left(\frac{dv_T}{dx}-\frac{dv_N}{dx}\right) \tag{48}$$

When there is no entry cost shock, the sign of the variation of relative number of varieties between two sector is crucially depends on ρ. Consider a case where the traded basket becomes cheaper compared to that of nontraded. Under $0 < \rho < 1$ and higher preference on nontraded goods ($\delta < 1/2$), a weak substitution effect and relatively strong negative income effect, causing a not enough relative increase of the demand of tradable basket, decreases the relative spending on tradables. The relative profits between two sectors also decreases one to one inducing the exist in the traded and entry in the nontraded sector.[10] When $1 < \rho \leq \sigma$, this high elasticity of substitution is enough to increase the relative spending on traded goods which translates into a positive variation of

[8] The substitution effects are found by diriving the Hicksien demand for both type of basket :

$$C_T = \frac{\delta}{\left[\delta+(1-\delta)\left(\frac{P_T}{P_N}\right)^{\rho-1}\right]^{\frac{1}{1-\frac{1}{\rho}}}}C \tag{45}$$

$$C_N = \frac{(1-\delta)}{\left[\delta\left(\frac{P_T}{P_N}\right)^{1-\rho}+(1-\delta)\right]^{\frac{1}{1-\frac{1}{\rho}}}}C \tag{46}$$

where C is the orignial consumption bundlle.

[9] Observe also if there is no nontraded sector ($\delta = 1$ impliying $\lambda = \infty$) the spending on traded sector is constant. Hence there is no variation of the profit.

[10] The variation of the number of varieties take an opposite sign between two sector in case of marginal cost shock. See Appendix 1.

the relative number of firms in traded sector. However this rich demand side effect disappears when $\rho = 1$. The same argument holds for Foreign varieties.[11]

In table 1 we report our numerical results with different value of ρ. Our choice of parameters, ρ (= 0.5, 1 and 2) and δ (= 0.25) comes from Obstfeld and Rogoff (2005). We set $\tau = 0.5$ which is used as a base line parameter in CMP (2006). Also the simulation is done for two different value of σ (= 3 and 6) which are broadly consistent with the literature of a macro and a micro empirical estimation. Each number in the table means the elasticity from the steady state value.

With a marginal cost shock in tradable sector, the extensive margins appears under $\rho \neq 1$. For both value of σ, with $\rho = 0.5$, Home tradalbe varieties die and its nontradable born because of a demand side effect discussed above. Under our symmetric parametrization, the same pattern of variation happens at Foreign but a lower magnitude. This important death of Home tradable varieties relative to Foreign makes the relative wage appreciation weaker and TOT depreciation stronger compared to the benchmark Cobb-Douglas case. As a result empirical based RER show a smaller appreciation reflecting this stronger TOT depreciation. The transmission from this TOT movement becomes more important for empirically consistent Foreign consumption (a higher variation of $\widetilde{C}^*$ relative to the Cobb-Douglas case). However in welfare based the gains for Foreigners becomes smaller reflecting a smaller range of available Home varieties (a lower variation of C^* relative to the Cobb-Douglas case)

Under $\rho = 2$, the sign of the variation of varieties between two sector is reversed. The same pattern takes place at Foreign with a lower magnitude. New creations of varieties at Home tradable sector makes the relative wage more appreciated. This appears as a weaker depreciation of TOT relative to the benchmark. At an extreme case (with $\sigma = 3$) the sign of TOT is even reversed. The empirical based RER shows a higher appreciation reflecting this weaker TOT depreciation and a stronger appreciation of the Home nontradables relative to Foreign. The transmission to Foreign becomes more important in welfare based with a larger range of available imported varieties. This contrasts with the case of empirical based Foreign consumption which increases less (because of a weaker positive transmission which stems from a weaker TOT depreciation) relative to the benchmark case.

With an entry cost shock on Home tradable sector, under $\rho \neq 1$, in addition

[11] From the definition of price indices noting:

$$\frac{1}{\overline{P_T}}\frac{dP_T}{dx} = \frac{1}{1+\phi}\frac{1}{\overline{p_T}}\frac{dp_T}{dx} + \frac{\phi}{1+\phi}\left(\frac{1}{\overline{\varepsilon}}\frac{d\varepsilon}{dx} + \frac{1}{\overline{p_T^*}}\frac{dp_T^*}{dx}\right) - \frac{1}{\sigma-1}\frac{1}{1+\phi}\left[\frac{1}{\overline{n_T}}\frac{dn_T}{dx} + \phi\frac{1}{\overline{n_T}}\frac{dn_T^*}{dx}\right] \tag{49}$$

$$\frac{1}{\overline{P_N}}\frac{dP_N}{dx} = \frac{1}{\overline{p_N}}\frac{dp_N}{dx} - \frac{1}{\sigma-1}\frac{1}{\overline{n_N}}\frac{dn_N}{dx} \tag{50}$$

In fact in our general equilibrium framework the variations of the number of varieties, endogenously determined, depends on themselves and other endogenously determined variable, ε which are in the price indices.

to the supply side effect, there is also that of from demand side.[12] For both value of σ, under the low elasticity of substitution, less Home varieties appears compared to the benchmark case and the positive extensive margins appears at Home nontradable sector. Reflecting the same type of demand side effect but with a lower magnitude, the number of Foreign tradables decreases and its nontradables increases. With $\rho = 2$ the effect is symmetric. More Home (Foreign) tradable varieties and less Home (Foreign) nontradables appears. For all value of ρ the TOT appreciates perfectly reflecting a real wage appreciation because of the important positive extensive margins in Home traded sector.

In case of the shock on Home nontradalbe sector, we observe a symmetrical results on the number of varieties compared to Home tradable sector's shocks. With a marginal cost shock on nontradable sector under low elasticity of substitution, the number of Home (Foreign) nonradable decreases and its tradable increases. Under high elasticity the number of Home (Foreign) nontradables increases and its tradables decreases. With an entry cost shock on notradable sector, less varieties in case of low elasticity and more varieties in case of high elasticity appear at Home nontradable sector. These extensive margins create an additional effect on the international prices, hence on the international transmission, both empirical and welfare based compared to the benchmark Cobb-Douglas case.

Conclusion

Our main findings are as follows: Except the Cobb-Douglas case, a non unity elasticity of substitution between traded and nontraded consumption bundle creates an additional variation of relative spending which falls on two sectors. Under a low elasticity a positive productivity shock at traded sector doesn't necessary means an expansion of the entry in that sector. This is the case where a decrease of the price of traded goods doesn't be accompanied by an enough increase of the corresponding demand because of the existence of competing nontraded goods. This endogenously arising demand side effect results in a variation of relative profits between traded and nontraded sector, which in turn induces the entry and exit of variety representing firms in both sectors. Because together with the intensive margins the degree of the extensive margins in the international market determines the degree of the international relative prices, this additional effect in terms of extensive margins amplifies or moderates the movement of theses international relative prices. One of the next challenges would be the extension of this static framework to the dynamic one which contains the investment decision.

[12] The demand side effect happens because the welfare based price indices change along with the variation of the number of varieties creating the variation of the relative spending between two sectors.

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Table 1

The simulation is done with $\delta = 0.25$ and $\tau = 0.5$ for all type of shocks

	$\sigma = 6$			$\sigma = 3$		
$d\alpha_T > 0$	$\rho = 0.5$	$\rho = 1$	$\rho = 2$	$\rho = 0.5$	$\rho = 1$	$\rho = 2$
n_T	−0.3308	0	0.9661	−0.2804	0	1.6296
n_N	0.1205	0	−0.2524	0.1136	0	−0.2049
n_T^*	−0.0024	0	0.0250	−0.0043	0	0.1470
n_N^*	0.0009	0	−0.0065	0.0017	0	−0.0185
ε	−0.8727	−0.8983	−0.9718	−0.7327	−0.7795	−1.0303
TOT	0.1273	0.1017	0.0282	0.2673	0.2205	−0.0303
P_T	−0.9267	−0.9882	−1.1680	−0.8448	−0.9743	−1.7321
P_N	−0.0241	0	0.0505	−0.0568	0	0.1025
P_T^*	−0.0067	−0.0118	−0.0302	−0.0129	−0.0257	−0.1562
P_N^*	−0.0002	0	0.0013	−0.0009	0	0.0092
RER	−0.6096	−0.6543	−0.7751	−0.4531	−0.5423	−0.9371
C	0.2651	0.2470	0.2019	0.2840	0.2436	0.1025
C^*	0.0019	0.0030	0.0052	0.0043	0.0064	0.0092
$\widetilde{RER}$	−0.6136	−0.6543	−0.7660	−0.4624	−0.5423	−0.9179
$\widetilde{C}$	0.2630	0.2470	0.2064	0.2793	0.2436	0.1121
$\widetilde{C^*}$	0.0040	0.0030	0.0007	0.0090	0.0064	−0.0004

	$\sigma = 6$			$\sigma = 3$		
$dv_T > 0$	$\rho = 0.5$	$\rho = 1$	$\rho = 2$	$\rho = 0.5$	$\rho = 1$	$\rho = 2$
n_T	0.9338	1.0000	1.1932	0.8598	1.0000	1.8148
n_N	0.0241	0	−0.0505	0.0568	0	−0.1025
n_T^*	−0.0005	0	0.0050	−0.0021	0	0.0735
n_N^*	0.0002	0	−0.0013	0.0009	0	−0.0092
ε	−0.1745	−0.1797	−0.1944	−0.3664	−0.3897	−0.5152
TOT	−0.1745	−0.1797	−0.1944	−0.3664	−0.3897	−0.5152
P_T	−0.1853	−0.1976	−0.2336	−0.4224	−0.4872	−0.8660
P_N	−0.0048	0	0.0101	−0.0284	0	0.0512
P_T^*	−0.0013	−0.0024	−0.0060	−0.0064	−0.0128	−0.0781
P_N^*	−0.0000	0	0.0003	−0.0004	0	0.0046
RER	−0.1219	−0.1309	−0.1550	−0.2266	−0.2712	−0.4686
C	0.0530	0.0494	0.0404	0.1420	0.1218	0.0512
C^*	0.0004	0.0006	0.0010	0.0022	0.0032	0.0046
$\widetilde{RER}$	−0.1637	−0.1692	−0.1850	−0.3418	−0.3671	−0.5018
$\widetilde{C}$	0.0054	0.0052	0.0047	0.0123	0.0113	0.0067
$\widetilde{C^*}$	−0.0054	−0.0052	−0.0047	−0.0123	−0.0113	−0.0067

	$\sigma = 6$			$\sigma = 3$		
$d\alpha_N > 0$	$\rho = 0.5$	$\rho = 1$	$\rho = 2$	$\rho = 0.5$	$\rho = 1$	$\rho = 2$
n_T	0.3348	0	−0.9775	0.2879	0	−1.6697
n_N	−0.1219	0	0.2554	−0.1166	0	0.2100
n_T^*	−0.0016	0	−0.0136	−0.0032	0	−0.1069
n_N^*	0.0006	0	0.0035	0.0013	0	0.0134
ε	−0.0262	0	0.0752	−0.0492	0	0.2644
TOT	−0.0262	0	0.0752	−0.0492	0	0.2644
P_T	−0.0622	0	0.1818	−0.1327	0	0.7747
P_N	−0.9756	−1.0000	−1.0511	−0.9417	−1.0000	−1.1050
P_T^*	−0.0045	0	0.0164	−0.0096	0	0.1136
P_N^*	−0.0001	0	−0.0007	−0.0006	0	−0.0067
RER	0.7042	0.7500	0.8737	0.6560	0.7500	1.1662
C	0.7317	0.7500	0.7957	0.7085	0.7500	0.8950
C^*	0.0013	0	−0.0028	0.0032	0	−0.0067
$\widetilde{RER}$	0.7084	0.7500	0.8644	0.6658	0.7500	1.1459
$\widetilde{C}$	0.7338	0.7500	0.7911	0.7133	0.7500	0.8849
$\widetilde{C^*}$	−0.0008	0	0.0018	−0.0017	0	0.0034

	$\sigma = 6$			$\sigma = 3$		
$dv_N > 0$	$\rho = 0.5$	$\rho = 1$	$\rho = 2$	$\rho = 0.5$	$\rho = 1$	$\rho = 2$
n_T	0.0670	0	−0.1955	0.1439	0	−0.8349
n_N	0.9756	1.0000	1.0511	0.9417	1.0000	1.1050
n_T^*	−0.0003	0	−0.0027	−0.0016	0	−0.0534
n_N^*	0.0001	0	0.0007	0.0006	0	0.0067
ε	−0.0052	0	0.0150	−0.0246	0	0.1322
TOT	−0.0052	0	0.0150	−0.0246	0	0.1322
P_T	−0.0124	0	0.0364	−0.0664	0	0.3874
P_N	−0.1951	−0.2000	−0.2102	−0.4708	−0.5000	−0.5525
P_T^*	−0.0009	0	0.0033	−0.0048	0	0.0568
P_N^*	−0.0000	0	−0.0001	−0.0003	0	−0.0034
RER	0.1408	0.1500	0.1747	0.3280	0.3750	0.5831
C	0.1463	0.1500	0.1591	0.3542	0.3750	0.4475
C^*	0.0003	0	−0.0006	0.0016	0	−0.0034
$\widetilde{RER}$	−0.0049	0	0.0143	−0.0230	0	0.1288
$\widetilde{C}$	0.0002	0	−0.0004	0.0008	0	−0.0017
$\widetilde{C^*}$	−0.0002	0	0.0004	−0.0008	0	0.0017

Appendix 1 : Solution for the number of varieties of each sector

Labor market clearing condition at steady state such as:

$$\sigma(\overline{n_T} + \overline{n_N}) = 1 \tag{51}$$

With (39), we get:

$$\overline{n_T} = \frac{1}{\sigma}\left[\left(\frac{1-\delta}{\delta}\right)^{\frac{\sigma-1}{\sigma-\rho}}\left(\frac{1}{1+\phi}\right)^{\frac{\rho-1}{\sigma-\rho}} + 1\right]^{-1}, \qquad \overline{n_N} = \frac{1}{\sigma}\left[\left(\frac{\delta}{1-\delta}\right)^{\frac{\sigma-1}{\sigma-\rho}}(1+\phi)^{\frac{\rho-1}{\sigma-\rho}} + 1\right]^{-1} \tag{52}$$

The steady state number of varieties is a function of love for variety. An increase of σ increases the number of varieties in both sector. In the case of $\rho = 1$, the number of varieties of each sector is given by :

$$\overline{n_T} = \frac{1}{\sigma}\delta, \qquad \overline{n_N} = \frac{1}{\sigma}(1-\delta) \tag{53}$$

In the same way we can derive the explicit variation of each sector's number of varieties. For any type of shock dx, totally differentiating the labor market clearing condition at home we get:

$$\lambda\frac{1}{\overline{n_T}}\frac{dn_T}{dx} + \frac{1}{\overline{n_N}}\frac{dn_N}{dx} = \lambda\frac{dv_T}{dx} + \frac{dv_N}{dx} \tag{54}$$

with (42) we have:

$$\frac{1}{\overline{n_T}}\frac{dn_T}{dx} = -(\rho-1)\left(\frac{\sqrt{2}}{\lambda+1}\right)^2\left(\frac{1}{\overline{P_T}}\frac{dP_T}{dx} - \frac{1}{\overline{P_N}}\frac{dP_N}{dx}\right) + \frac{dv_T}{dx} \tag{55}$$

$$\frac{1}{\overline{n_N}}\frac{dn_N}{dx} = (\rho-1)\left(\frac{\sqrt{2\lambda}}{\lambda+1}\right)^2\left(\frac{1}{\overline{P_T}}\frac{dP_T}{dx} - \frac{1}{\overline{P_N}}\frac{dP_N}{dx}\right) + \frac{dv_N}{dx} \tag{56}$$

The variation of the number of varieties in each sector move in opposite direction in case of marginal cost shock which changes the relative price between tradable and nontradable basket. The similar expressions hold for Foreign varieties.

Appendix 2 : Analytical result under $\rho \neq 1$

Setting $\rho = 1$, we find the table in section 4. Also we can get the same result as CMP (2006) setting $\lambda = \infty$ (no nontradables at steady state) for inelastic labor supply and Dixit-Stiglitz preference case.

Shock $d\alpha_T > 0$

$$\frac{1}{n_T}\frac{dn_T}{d\alpha_T} = \frac{(\sigma-1)(\rho-1)\left(\frac{1}{1+\lambda}\right)}{\Delta}\left[2\sigma-1+\sigma\phi\frac{\rho-1}{\sigma-\rho}\left(\frac{1}{1+\lambda}\right)\right] \tag{57}$$

$$\frac{1}{n_N}\frac{dn_N}{d\alpha_T} = -\frac{(\sigma-1)(\rho-1)\left(\frac{\lambda}{1+\lambda}\right)}{\Delta}\left[2\sigma-1+\sigma\phi\frac{\rho-1}{\sigma-\rho}\left(\frac{1}{1+\lambda}\right)\right] \tag{58}$$

$$\frac{1}{n_T}\frac{dn_T^*}{d\alpha_T} = \frac{(\sigma-1)(\rho-1)\left(\frac{1}{1+\lambda}\right)}{\Delta}\phi\left[1+\sigma\frac{\rho-1}{\sigma-\rho}\left(\frac{1}{1+\lambda}\right)\right] \tag{59}$$

$$\frac{1}{n_N}\frac{dn_N^*}{d\alpha_T} = -\frac{(\sigma-1)(\rho-1)\left(\frac{\lambda}{1+\lambda}\right)}{\Delta}\phi\left[1+\sigma\frac{\rho-1}{\sigma-\rho}\left(\frac{1}{1+\lambda}\right)\right] \tag{60}$$

$$\frac{1}{\varepsilon}\frac{d\varepsilon}{d\alpha_T} = -\frac{(\sigma-1)\left[2(\sigma-\rho)+(\rho-1)(1+\phi)\left(\frac{1}{1+\lambda}\right)\right]}{\Delta} < 0 \tag{61}$$

where

$$\Delta \equiv (\sigma-\rho)(2\sigma-1+\phi)+2\sigma(\rho-1)\phi\left(\frac{1}{1+\lambda}\right) > 0^{13} \tag{62}$$

Shock $dv_T > 0$

$$\frac{1}{n_T}\frac{dn_T}{dv_T} = 1+\frac{(\rho-1)\left(\frac{1}{1+\lambda}\right)}{\Delta}\left[2\sigma-1+\sigma\phi\frac{\rho-1}{\sigma-\rho}\left(\frac{1}{1+\lambda}\right)\right] \tag{63}$$

$$\frac{1}{n_N}\frac{dn_N}{dv_T} = -\frac{(\rho-1)\left(\frac{\lambda}{1+\lambda}\right)}{\Delta}\left[2\sigma-1+\sigma\phi\frac{\rho-1}{\sigma-\rho}\left(\frac{1}{1+\lambda}\right)\right] \tag{64}$$

$$\frac{1}{n_T}\frac{dn_T^*}{dv_T} = \frac{(\rho-1)\left(\frac{1}{1+\lambda}\right)}{\Delta}\phi\left[1+\sigma\frac{\rho-1}{\sigma-\rho}\left(\frac{1}{1+\lambda}\right)\right] \tag{65}$$

[13] Note for a reminder:
$0 \leq \frac{1}{1+\lambda} \leq 1$ and $0 \leq \frac{\lambda}{1+\lambda} \leq 1$

$$\frac{1}{n_N}\frac{dn_N^*}{dv_T} = -\frac{(\rho-1)\left(\frac{\lambda}{1+\lambda}\right)}{\Delta}\phi\left[1+\sigma\frac{\rho-1}{\sigma-\rho}\left(\frac{1}{1+\lambda}\right)\right] \tag{66}$$

$$\frac{d\varepsilon}{dv_T} = -\frac{2(\sigma-\rho)+(\rho-1)(1+\phi)\left(\frac{1}{1+\lambda}\right)}{\Delta} < 0 \tag{67}$$

<u>Shock $d\alpha_N > 0$</u>

$$\frac{1}{n_T}\frac{dn_T}{d\alpha_N} = -\frac{(\sigma-1)(\rho-1)\left(\frac{1}{1+\lambda}\right)}{\Delta}\left[2\sigma-1+\phi+\sigma\phi\frac{\rho-1}{\sigma-\rho}\left(\frac{1}{1+\lambda}\right)\right] \tag{68}$$

$$\frac{1}{n_N}\frac{dn_N}{d\alpha_N} = \frac{(\sigma-1)(\rho-1)\left(\frac{\lambda}{1+\lambda}\right)}{\Delta}\left[2\sigma-1+\phi+\sigma\phi\frac{\rho-1}{\sigma-\rho}\left(\frac{1}{1+\lambda}\right)\right] \tag{69}$$

$$\frac{1}{n_T}\frac{dn_T^*}{d\alpha_N} = -\frac{(\sigma-1)(\rho-1)\left(\frac{1}{1+\lambda}\right)}{\Delta}\left[\sigma\phi\frac{(\rho-1)}{\sigma-\rho}\left(\frac{1}{1+\lambda}\right)\right] \tag{70}$$

$$\frac{1}{n_N}\frac{dn_N^*}{d\alpha_N} = \frac{(\sigma-1)(\rho-1)\left(\frac{\lambda}{1+\lambda}\right)}{\Delta}\left[\sigma\phi\frac{(\rho-1)}{\sigma-\rho}\left(\frac{1}{1+\lambda}\right)\right] \tag{71}$$

$$\frac{d\varepsilon}{d\alpha_N} = \frac{(\sigma-1)(\rho-1)(1-\phi)\left(\frac{1}{1+\lambda}\right)}{\Delta} \tag{72}$$

<u>Shock $dv_N > 0$</u>

$$\frac{1}{n_T}\frac{dn_T}{dv_N} = -\frac{(\rho-1)\left(\frac{1}{1+\lambda}\right)}{\Delta}\left[2\sigma-1+\phi+\sigma\phi\frac{\rho-1}{\sigma-\rho}\left(\frac{1}{1+\lambda}\right)\right] \tag{73}$$

$$\frac{1}{n_N}\frac{dn_N}{dv_N} = 1+\frac{(\rho-1)\left(\frac{\lambda}{1+\lambda}\right)}{\Delta}\left[2\sigma-1+\phi+\sigma\phi\frac{\rho-1}{\sigma-\rho}\left(\frac{1}{1+\lambda}\right)\right] \tag{74}$$

$$\frac{1}{n_T}\frac{dn_T^*}{dv_N}=-\frac{(\rho-1)\left(\frac{1}{1+\lambda}\right)}{\Delta}\left[\sigma\phi\frac{(\rho-1)}{\sigma-\rho}\left(\frac{1}{1+\lambda}\right)\right] \tag{75}$$

$$\frac{1}{n_N}\frac{dn_N^*}{dv_N}=\frac{(\rho-1)\left(\frac{\lambda}{1+\lambda}\right)}{\Delta}\left[\sigma\phi\frac{(\rho-1)}{\sigma-\rho}\left(\frac{1}{1+\lambda}\right)\right] \tag{76}$$

$$\frac{d\varepsilon}{dv_N}=\frac{(\rho-1)(1-\phi)\left(\frac{1}{1+\lambda}\right)}{\Delta} \tag{77}$$

For any type of shock dx the variation of price indices is found using the relationships below (note also for any price index the "empirical" version is obtained by eliminating the terms which represent the variation of the number of varieties).:

$$TOT\equiv\frac{\varepsilon p_T^*}{p_T} \tag{78}$$

$$\frac{1}{TOT}\frac{dTOT}{dx}=\frac{1}{\varepsilon}\frac{d\varepsilon}{dx}+\frac{1}{p_T^*}\frac{dp_T^*}{dx}-\frac{1}{p_T}\frac{dp_T}{dx} \tag{79}$$

$$\frac{1}{RER}\frac{dRER}{dx}=\frac{1}{\varepsilon}\frac{d\varepsilon}{dx}+\frac{1}{P}\frac{dP^*}{dx}-\frac{1}{P}\frac{dP}{dx} \tag{80}$$

for Home price indices,

$$\frac{1}{P}\frac{dP}{dx}=\frac{\lambda}{\lambda+1}\frac{1}{P_T}\frac{dP_T}{dx}+\frac{1}{\lambda+1}\frac{1}{P_N}\frac{dP_N}{dx} \tag{81}$$

$$\frac{1}{P_T}\frac{dP_T}{dx}=\frac{1}{1+\phi}\frac{1}{p_T}\frac{dp_T}{dx}+\frac{\phi}{1+\phi}\left(\frac{1}{\varepsilon}\frac{d\varepsilon}{dx}+\frac{1}{p_T^*}\frac{dp_T^*}{dx}\right)-\frac{1}{\sigma-1}\frac{1}{1+\phi}\left[\frac{1}{n_T}\frac{dn_T}{dx}+\phi\frac{1}{n_T}\frac{dn_T^*}{dx}\right] \tag{82}$$

$$\frac{1}{P_N}\frac{dP_N}{dx}=\frac{1}{p_N}\frac{dp_N}{dx}-\frac{1}{\sigma-1}\frac{1}{n_N}\frac{dn_N}{dx} \tag{83}$$

for Foreign price indices,

$$\frac{1}{P}\frac{dP^*}{dx}=\frac{\lambda}{\lambda+1}\frac{1}{P_T^*}\frac{dP_T^*}{dx}+\frac{1}{\lambda+1}\frac{1}{P_N^*}\frac{dP_N^*}{dx} \tag{84}$$

$$\frac{1}{P^*}\frac{dP_T^*}{dx}=\frac{1}{1+\phi}\frac{1}{p_T^*}\frac{dp_T^*}{dx}+\frac{\phi}{1+\phi}\left(-\frac{1}{\varepsilon}\frac{d\varepsilon}{dx}+\frac{1}{p_T}\frac{dp_T}{dx}\right)-\frac{1}{\sigma-1}\frac{1}{1+\phi}\left[\frac{1}{n_T^*}\frac{dn_T^*}{dx}+\phi\frac{1}{n_T}\frac{dn_T}{dx}\right] \tag{85}$$

$$\frac{1}{P^*}\frac{dP_N^*}{dx} = \frac{1}{p_N^*}\frac{dp_N^*}{dx} - \frac{1}{\sigma-1}\frac{1}{n_N^*}\frac{dn_N^*}{dx} \tag{86}$$

Appendix 3: The simplest HBS effect

For any type of shock, the empirical based HBS effect is captured as:

$$\frac{1}{\overline{RER}}\frac{d\widetilde{RER}}{dx} = \frac{1-\phi}{1+\phi}\frac{\lambda}{\lambda+1}\frac{1}{\overline{TOT}}\frac{dTOT}{dx} + \frac{1}{\lambda+1}\left(\frac{1}{\overline{\varepsilon}}\frac{d\varepsilon}{dx} + \frac{1}{\overline{p_N^*}}\frac{dp_N^*}{dx} - \frac{1}{\overline{p_N}}\frac{dp_N}{dx}\right) \tag{87}$$

Under $\rho = 1$ and with a marginal cost shock at Home tradable sector,

$$\frac{1}{\widetilde{RER}}\frac{d\widetilde{RER}}{d\alpha_T} = \frac{1-\phi}{1+\phi}\delta\frac{1+\phi}{2\sigma-1+\phi}+(1-\delta)\left(-\frac{2(\sigma-1)}{2\sigma-1+\phi}\right) = \frac{(1-\phi)\delta-2(\sigma-1)(1-\delta)}{2\sigma-1+\phi} \tag{88}$$

The sign of the variation is ambiguous. Imposing $\phi = 1$ (no trade cost) we have:

$$\frac{1}{\widetilde{RER}}\frac{d\widetilde{RER}}{d\alpha_T} = -\left(\frac{\sigma-1}{\sigma}\right)(1-\delta) < 0 \tag{89}$$

Now this economy exhibits an unambiguous real appreciation however somewhat a moderated way because of the term which comes from the fact of product differentiation, $0 < \dfrac{\sigma-1}{\sigma} < 1$. Imposing $\sigma = \infty$ we get :

$$\frac{1}{\widetilde{RER}}\frac{d\widetilde{RER}}{d\alpha_T} = -(1-\delta) < 0 \tag{90}$$

The real appreciation becomes stronger with more spending weight on nontraded goods. Here we refined the HBS effect in a very simple (and easy) form where no international transmission takes place (no TOT movement).