Monetary Policy and National Divergences in a Heterogeneous Monetary Union

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Abstract

It is widely recognized that the Euro area is an asymmetric monetary Union which assembles countries with heterogeneous structures on financial, goods and labor markets stricken by asymmetric shocks. However, the main objective of the European Central Bank (ECB) is to preserve price stability for the euro area as a whole, and the ECB pays most of its attention to Union-wide output and (principally) inflation, neglecting, at least on the level of principles, inflation and output divergences in Union. In this paper, we wonder, at a theoretical level, about the social loss associated with such an objective based on aggregate magnitudes, and we search for solutions, namely an "optimal" contract for a common central bank. We show in particular that it is not necessarily a good thing that a common central bank worries about inflation divergences without being concerned about output divergences in Union.

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Should a common central bank in a heterogeneous monetary Union consider national divergences, and how should she do it? This question is at the heart of the monetary policy matter in the Euro area. It is widely recognized that the Euro area is an asymmetric monetary Union which assembles countries with heterogeneous structures on financial, goods and labor markets stricken by asymmetric shocks. However, the main objective of the European Central Bank (ECB) is to preserve price stability for the euro area as a whole, and the ECB pays most of its attention to Union-wide output and (principally) inflation, neglecting, at least on the level of principles, inflation and output divergences in Union. In this paper, we wonder, at a theoretical level, about the social loss associated with such an objective based on aggregate magnitudes, and we search for solutions, namely an "optimal" contract for a common central bank.

Inflation and output-growth divergences in the Euro area are well documented[1]. In 2005, for instance, the degree of inflation and of output dispersion, measured as the un-weighted standard deviation among the 12 EMU countries was respectively about 0.86 and 1.5 percentage points. These values reflect very different inflation and output-gap positions in the area (see *Table 1*). Moreover, the enlargement of EMU is likely to emphasize the relative size of output-gap and inflation divergences, since the new participants' relatively low level of economic development will structurally produce higher than average output growth and inflation rates during the catching-up path (through a Balassa-Samuelson effect). Enlargement of the EMU will also probably increase the heterogeneity in the transmission channel of the common monetary policy, since the new participants experience different processes of financial liberalization.

Table 1: Inflation and output-gap in EMU

Year	*1999*	*2000*	*2001*	*2002*	*2003*	*2004*	*2005*
INFLATION	*0.72*	*1.03*	*1.06*	*1.14*	*0.97*	*0.85*	*0.86*
OUTPUT-GAP	*2.52*	*2.06*	*1.67*	*1.77*	*1.84*	*1.29*	*1.5*

Un-weighted standard deviation of inflation and output-gap. Source: OECD.

A number of recent theoretical papers have shown that asymmetries in the transmission of monetary policy call for a design of monetary policies that consider national data and not only average or aggregate (EMU) data. The preeminence of strategies based upon national data has notably been established by De Grauwe (2000), Gros & Hefeker (2002) and De Grauwe & Senegas (2003)[2]. Despite these results, the monetary policy design in the Euro area was originally mostly (and perhaps exclusively) based upon aggregate magnitudes[3], although a great part of the recent debate in monetary policy focus upon the implications of inflation differentials for the formulation of monetary policy (with few or no reference to output differentials).

In this paper, we construct a theoretical model to assess the benefits of a common central bank that worries about inflation and/or output differentials in a monetary Union. We model a simple n-country monetary Union in which monetary policy is designed by a

[1] See, for example, Altissimo, Angeloni & Ehrmann (2004), Musso & Westermann (2005) and the analysis of the ECB (2005) of the consequences of these divergences for the conduct of monetary policy.

[2] De Grauwe & Sénégas (2003) find that uncertainty reinforces the case for using a national perspective.

[3] For example, in the Governing Council congregation of September, 09, 1999, President Duisenberg asserted: *"…our decisions today, again and as always, were based on a euro area-wide analysis (…) –and nothing else".* This point is strengthened by the declaration following the Governing Council meeting dated from March, 30 (2000): *"as laid down in the Treaty, each Member of the Governing Council is therefore well aware that he or she is not a representative of a country (…) but acts (…) in deciding the appropriate conduct of monetary policy for the euro area as a whole".*

common central bank that is only concerned about average magnitudes (inflation and output-gap). The central bank possesses its own loss function (that we call "centralized" loss function), which differs from the Union loss function, which is the average of national loss functions (that we call "cooperative" loss function). What is the cost of this "wrong" objective, in terms of social welfare?

In a homogenous Union, without "structural" heterogeneity neither in the transmission process of monetary policy or in the objective defended by the Member-States of Union, the difference between the two loss functions would be irrelevant: using a centralized loss function or a cooperative one would give rise to the same solution, even if countries are stricken by idiosyncratic shocks. In such a homogenous Union, neglecting regional inflation or output differentials would involve no cost.

Thus, in order to deal with costs associated with a centralized policymaking, compared with a cooperative one, one has to consider some degree of "structural" heterogeneity in the Union. If the transmission channel of monetary policy is asymmetric, for example, a centralized policymaking is less efficient, from the Union-wide welfare point of view, than a "cooperative" one, a result established by Gros & Hefeker (2002) and De Grauwe & Senegas (2003).

In the present model, we introduce structural heterogeneity in the simplest way, namely in the transmission channel of the common interest rate to aggregate demand. Our model neglects other sources of heterogeneity in Union, such as labor market heterogeneity (in link with divergences in wage setting) or goods market heterogeneity (in link with differences in cyclical positions, economic development levels or openness degrees, for example); not because these sources of heterogeneity are less significant, but because they exceed the pure monetary transmission dimension. Furthermore, the heterogeneity of the transmission channel of monetary policy is a direct clause of concern for the ECB[4]. In addition to this "structural" asymmetry, we introduce idiosyncratic supply and demand shocks. Thus, in some sense, our paper can be viewed as an extension of Gros & Hefeker (2002) and De Grauwe & Senegas (2006), with a precise modeling of asymmetry in the monetary transmission process and in the shocks that affect the different countries, and in which we explicitly study the optimal contract for the common central bank[5].

Our model shows that, under general assumptions, the inefficiency associated with a "centralized" monetary policy design (relative to a "cooperative" one) can be removed by setting an "optimal" contract for the central bank. This optimal contract penalizes the common central bank from inflation and output divergences in the Union. We show that the form of this optimal contract is very simple: the penalties imposed on inflation (respectively on output) divergences correspond to the relative weight of inflation (respectively output) in the social welfare function. The interpretation of the optimal contract is straightforward. If the common central bank is adequately penalized from inflation and output differentials, monetary policy will take account of the particular situation of each country, and will reach the Union-wide first best.

However, this result must receive some qualifications. First, the optimal contract is not beneficial to all countries of the Union. Therefore, setting a contract for the common central

[4] For example, in EMU countries, the relative size of the "credit channel" or the "interest channel" of monetary policy may produce divergent effects of monetary policy impulses. See in particular, Issing & al. (2001) and Mojon & Peersman (2001). Moreover, the enlargement of EMU will probably heighten uncertainty about the transmission channels (Hefeker, 2004).

[5] Gros & Hefeker (2002) and De Grauwe & Senegas (2006) only consider symmetric shocks, but, as we shall see, the mix between symmetric and asymmetric shocks strongly affects the form of the central bank contract.

bank may be a source of conflicts between the Member-States, even if this contract is optimal from the point of view of Union-wide welfare.

Second, optimal penalties take simple values only if the different Member-States and the common central bank share the same relative preferences for output and inflation stabilization. In the opposite case, one can generally still find an optimal contract, but this contract is more complicated, and its penalties become model-dependent. If the common central bank is more concerned with the stabilization of inflation than output, relative to Union preferences, for example, our model shows that, in the optimal contract, the penalties imposed on inflation (respectively on output) divergences are higher than the relative weighs of inflation (respectively output) in the social welfare function and are inversely linked with the degree of heterogeneity in the Union.

Third, in the Euro area, the ECB is unlikely to focus on output differentials. Effectively, monetary policy reports of the Euro area more and more discusses the consequences of inflation differentials, but with few or no reference to output gap differentials. So, we turn our attention to a situation in which the common central bank is only concerned with inflation differentials. In such a case, no optimal contract can be implemented, but a "second best" contract can be found, which minimizes the Union-wide social loss function. Our model establishes notably that, if output divergences are not a clause of concern for the common central bank, the second best coefficient for inflation divergence is not necessarily positive. Thus, attempting to reduce inflations divergences in a heterogeneous monetary Union is not necessarily a good prescription, if this prescription is not supported by an output divergence-oriented device.

The paper is structured as follows. Section 1 introduces the main characteristics of the model. Besides demand and supply shocks, we introduce structural asymmetries on the interest rate transmission channel. Section 2 investigates the cost of a centralized monetary policy design, relative to the optimal "cooperative" solution. In section 3 we assess the optimal contract for the common central bank, and in section 4 and 5 we study respectively how the optimal contract has to be modified when the central bank does not share social preferences for the stabilization of output relative to inflation, and the form of "second best" contracts when she does not take output divergences into consideration.

1/ The model

Our model depicts a n-country closed monetary Union. All countries have the same size and are indexed by $i = 1,...,n$. Supply functions are defined by:

$$y_i^s = \alpha \pi_i + \mu_i^s \tag{1}$$

where π_i is the inflation rate and μ_i is a white noise supply shock with variance $\sigma^2_{\mu_i^s}$. All variables are specified in log-deviations (in particular, the natural level of output is zero, and all expected quantities are set to zero). Thus, relation (1) depicts a "Lucas supply function", in which equilibrium output can exceed natural product only when some "surprises" are present, either because of an exogenous supply shock or because of an inflation surprise which produces an ex post under-indexation of wages[6].

[6] Notice that, compared to Gros & Hefeker (2002) and De Grauwe & Senegas (2006); we suppose here that inflation rates may be different in the different countries. It is an important characteristic of our model, since we want to study the optimal way for the common central bank to take account of inflation divergences in the

In order to focus on heterogeneity in the Union, we specify very simple demand functions. In country i, demand depends on the Union-wide interest rate[7] (r), which is the monetary policy instrument set by the common central bank, on the inflation differential $(\pi - \pi_i)$, which depicts country i competitiveness ($\pi = \frac{1}{n}\sum_{i=1}^{n}\pi_i$ is the average inflation in the Union) and is affected by a white noise demand shock (μ_i^d) with variance $\sigma_{\mu_i^d}^2$:

$$y_i^d = \beta(\pi - \pi_i) - b_i r + \mu_i^d \tag{2}$$

In addition to supply and demand idiosyncratic shocks, we introduce some "structural" heterogeneity in the Union, and more precisely in the monetary policy transmission channel (b_i). In order to deal with « pure » heterogeneity effects, independently of average effects, we define coefficient b_i in deviation from its mean. Let $b = \frac{1}{n}\sum_{i=1}^{n} b_i$ be the average interest-elasticity of demand in the Union; we define $b_i = (1+\varepsilon_i)b$ as the country-i specific component of the monetary policy transmission channel, with $\varepsilon_i \in [-1,1]$ and $\sum_{i=1}^{n}\varepsilon_i = 0$.

In what follows, "structural" heterogeneity in the Union will be synthesized by the following coefficient: $\Sigma^2 \equiv \frac{1}{n}\sum_{i=1}^{n}\varepsilon_i^2 \in [0,1]$.

All shocks μ_i (and, more generally, all variables of the model) can be represented as the sum of an average component (μ), which affects every country in the same way, and a deviation from the mean component $(\bar{\mu}_i = \mu_i - \mu)$, specific to country i: $\mu_i \equiv \mu + \bar{\mu}_i$ (with of course: $\sum_{i=1}^{n}\bar{\mu}_i = 0$). In what follows, we will design as "symmetric" the component of shocks that affects every country in the same way (μ^s and μ^d), and as "asymmetric" the specific component of these shocks ($\bar{\mu}_i^s$ and $\bar{\mu}_i^d$).

To solve the model, we write equilibrium in average variables $(y^s = y^d)$ and in deviations[8] $(\bar{y}_i^s = \bar{y}_i^d)$. In *Appendix 1*, we show that equilibrium solutions are independent of coefficient b, so we can normalize this coefficient to: $b = 1$. We obtain the following symmetric (or average) and asymmetric (or specific) components of inflation:

$$\pi = \frac{\mu^d - \mu^s - r}{\alpha} \tag{3a}$$

Union. Thus, we cannot suppose, as these authors, that the central bank directly controls "the" inflation rate. In contrast, we must specify demand functions and study the monetary transmission process.

[7] Since expected inflation is zero, r denotes either nominal or real interest rate.

[8] Average supply and demand functions in the Union are: $y^s = \alpha\pi + \mu^s$ and: $y^d = \mu^d - r$; and, in deviation: $\bar{y}_i^s = \alpha\bar{\pi}_i + \bar{\mu}_i^s$ and: $\bar{y}_i^d = \bar{\mu}_i^d - \beta\bar{\pi}_i - \varepsilon_i r$.

$$\overline{\pi}_i = \frac{\overline{\mu}_i^d - \overline{\mu}_i^s - \varepsilon_i r}{\alpha + \beta}, \ \forall i \tag{3b}$$

and we can easily compute Union product, on average $\left(y = \frac{1}{n} \sum_{i=1}^{n} y_i \right)$ and in deviation:

$$y = \alpha\pi + \mu^s = \mu^d - r \tag{4a}$$

$$\overline{y}_i = \alpha\overline{\pi}_i + \overline{\mu}_i^s = \frac{\alpha\overline{\mu}_i^d - \alpha\varepsilon_i r + \beta\overline{\mu}_i^s}{\alpha + \beta}, \ \forall i \tag{4b}$$

In equation (4a), we can notice that Union average income does not depend on heterogeneity coefficients (ε_i). This is also the case for all average variables in Union. Equation (4b) shows that the transmission channel of the common interest rate is asymmetric, because of the heterogeneity of the Union.

We suppose that each country of the Union is endowed with a social loss function that depends on stabilization of income and inflation:

$$L_i = \frac{1}{2}\left[\lambda y_i^{\ 2} + \pi_i^{\ 2} \right] \tag{5}$$

where λ depicts social preferences for income stabilization relative to inflation stabilization. We also suppose that λ is the same in both countries, in order to focus on "structural" heterogeneity in the Union. The problem of heterogeneity of preferences in a monetary Union is an important, but distinct, question. Our model describes a Union where there are no preference conflicts, but simply differences in the functioning of economies.

Since all countries have the same size, the Union-wide social loss is:

$$L^U = \frac{1}{n} \sum_{i=1}^{n} L_i \tag{6}$$

In contrast with this social loss function, based on the average of national loss functions in the Union, we suppose that the common central bank chooses the Union-wide interest rate r, in order to minimize a loss function that depends on stabilization of income and inflation, *based on the average variables of the Union*:

$$L^C = \frac{1}{2}\left[\tilde{\lambda} y^2 + \pi^2 \right] \tag{7}$$

In the euro zone, for example, it is widely accepted that the decisions of the ECB are designed for minimizing an objective based on average euro variables rather than an objective made up of national loss functions. In our model, we depict such a situation by the fact that the common central bank minimizes a “centralized” loss function $\left(L^C\right)$ and not the Union-wide social loss function, which is a “cooperative” loss function $\left(L^U\right)$.

We first suppose that the common central bank shares the social preference parameter for the stabilization of output relative to inflation $\left(\tilde{\lambda}=\lambda\right)$, in order to focus on the impact of “centralized” versus “cooperative” designs of monetary policies. In section 4, we consider the alternative case in which the central bank possesses distinct preferences $\left(\tilde{\lambda}\neq\lambda\right)$

To keep the model simple, we also choose to focus exclusively on a stabilization problem for monetary policy, and we ignore possible systematic bias in monetary policy that emerges when the central bank defends an output target higher than the natural product (here zero). Such an inconsequent objective would lead to a well-known inflation bias, which can easily be removed by setting an optimal contract that penalizes the common central bank from inflation deviations. Walsh (1995) shows, in particular, that the optimal contract results in a linear penalty on inflation[9].

In our model, the minimization of (7) relative to (6) does not raise a problem of systematic bias, but, on the contrary, raises a stabilization problem for monetary policy. As a result, central bank preferences for the stabilization of output and inflation will have to be modified, by setting a kind of “quadratic” contract; since only quadratic contracts may affect the stabilization properties of monetary policies (such contracts are similar to changing preferences of central bankers as in Rogoff, 1985). The intuition of our results below is that penalties on inflation and output divergences are precisely a kind of quadratic contract.

2/ The cost of a centralized monetary policy

Let us now characterize the inefficiencies in monetary policy associated to the minimization of (7) rather than (6), considering first that the different Member States and the common central bank share the same relative preferences for output and inflation stabilization $\left(\tilde{\lambda}=\lambda\right)$.

The common central bank chooses its interest rate by minimizing (7), knowing the values of demand and supply shocks. The union-wide interest rate is thus (see *Appendix 1*):

$$r=r^c=\psi_1^c\mu^s+\psi_2^c\mu^d \tag{8}$$

where: $\psi_1^c=-1/\lambda_1$, $\psi_2^c=1$, and: $\lambda_1=1+\alpha^2\lambda$.

The optimal interest rate, obtained by minimizing (6) with respect to r is (see *Appendix 1*):

$$r=r^u=\psi_1^u\mu^s+\psi_2^u\mu^d+\psi_3^u\sum_{i=1}^{n}\left(\varepsilon_i\bar{\mu}_i^s\right)+\psi_4^u\sum_{i=1}^{n}\left(\varepsilon_i\bar{\mu}_i^d\right) \tag{9}$$

[9] In our model, if k is the output target of the common central bank, the optimal penalty for inflation deviations is: $c=\tilde{\lambda}k\alpha$, so that the common central bank minimizes: $L^C=\frac{1}{2}\left[\tilde{\lambda}\left(y-k\right)^2+\pi^2+2c\pi\right]$.

where: $\psi_1^u = \psi_1^c a_1 \lambda_1 / a$, $\psi_2^u = \psi_2^c a_1 \lambda_1 / a$, $\psi_3^u = \lambda_3 \alpha^2 / na$ and $\psi_4^u = \alpha^2 \lambda_1 / na$. All along the paper, we use the notations: $a_1 = (\alpha + \beta)^2$, $a_2 = \alpha^2 \Sigma^2$ and: $a = \lambda_1 (a_1 + a_2)$; as well as: $\lambda_1 = 1 + \alpha^2 \lambda$, $\lambda_2 = 1 + \beta^2 \lambda$ and $\lambda_3 = \alpha\beta\lambda - 1$.

A direct comparison between (8) and (9) allows identifying the inefficiencies in monetary policy. Results are summarized by the following *Proposition*:

Proposition 1

In a heterogeneous monetary Union, symmetric shocks have to be less stabilized and asymmetric shocks have to be more stabilized than in a homogenous monetary Union. If the common central bank minimizes a "centralized" loss function, by considering only average quantities in Union, its interest rate policy will involve an over-reaction to symmetric shocks and an insufficient reaction to asymmetric shocks.

Proof:

Concerning symmetric shocks, since: $a_1 \lambda_1 / a < 1$, we have: $|\psi_1^u| < |\psi_1^c|$ and: $\psi_2^u < \psi_2^c$ if $\Sigma^2 > 0$. On the one hand, the reaction of interest rate to symmetric supply shocks is excessive with a centralized monetary policy relative to a cooperative one. As a result, the Union-wide average product will be insufficiently stabilized in (4a), while average inflation will be too much stabilized in (3a). On the other hand, in the cooperative regime, demand shocks should be perfectly stabilized if the monetary Union were homogenous $(\Sigma^2 = 0 \Rightarrow \psi_2^u = 1)$, but have to be only partially stabilized in a heterogeneous Union (since $\psi_2^u < 1$ if $\Sigma^2 > 0$). Yet, with a centralized loss function, the common central bank continues completely stabilizing symmetric demand shocks, in spite of heterogeneity. As a result, average inflation and output in Union are to much stabilized, to the detriment of the stabilization of deviations ($\overline{y}_i$ and $\overline{\pi}_i$). These findings show that, concerning symmetric shocks, in a heterogeneous monetary Union, one needs a less reactive monetary policy than in a homogenous Union[10].

Concerning anti-symmetric shocks, by focusing on average variables of the Union, we can directly see from (10) and (11) that the common central bank does not take these shocks into account, while she should do under the optimal interest policy ($\psi_3^u \neq 0$ and $\psi_4^u \neq 0$). Thus, asymmetric shocks are not sufficiently stabilized in the Union. Average output and inflation are not affected, but the use of a centralized loss function increases divergences in the area: national quantities are not properly stabilized.

Notice that in a homogenous monetary Union $(\varepsilon_i = 0, \forall i)$, our model would give rise to the well-known equivalence between minimizing L^U or L^C. Thus, if there were no cross-country divergence, monetary authorities could rely on a loss function based on area wide variables only, without implying any welfare loss in Union. In a heterogeneous Union, on the contrary, the social loss will be higher with the interest rate rule (8) than with (9).

[10] This finding that can be compared to the « *precaution principle* » in models of monetary policy with uncertainty.

From the Union-wide welfare point of view, what matters is the *ex ante* (i.e. before knowing the value of shocks) value of the social loss function, namely EL^U where E denotes the rational expectation operator. To simplify the model, we suppose in the main text that there is no demand shock (we can for example suppose that these shocks are perfectly stabilized by fiscal policies). Effectively, the stabilization of demand shocks do not raise specific question and the way demand shocks are handled can be viewed as a special case of supply shocks analysis (see *Appendix 2*, which explicitly extends the analysis to demand shocks).

So, let us henceforth forget demand shocks, by setting from now: $\sigma^2_{\bar{\mu}_i^d} = \sigma^2_{\mu^d} = 0,\ \forall i$. We can respectively refer to: $\sigma^2_{\mu^s} = \sigma^2_{\mu}$ and $\sigma^2_{\bar{\mu}_i^s} = \sigma^2_{\bar{\mu}_i}$ for the variances of symmetric and asymmetric components of supply shocks. In addition, to save notations, we also suppose that: i) specific and average components of supply shocks are independently distributed, *i.e.* $\sigma_{\mu\bar{\mu}_i} = 0,\ \forall i$, ii) idiosyncratic supply shocks are independently distributed[11], *i.e.*: $\sigma_{\bar{\mu}_i\bar{\mu}_j} = 0,\ \forall i$, and iii) they have the same variance: $\sigma^2_{\bar{\mu}_i^s} = \sigma^2_{\bar{\mu}},\ \forall i$ (these assumptions are only notation-saving assumptions, with no generality loss; see *Appendix 1* for the general resolution of the model).

Therefore, under the optimal interest rate policy (9), the expected Union-wide social loss is:

$$EL^U\left(r^u\right) = X\sigma^2_{\mu} + \bar{X}\sigma^2_{\bar{\mu}} \tag{10a}$$

where: $X = \left(a - a_1\right)/2a\alpha^2$ and: $\bar{X} = \left(\lambda_2 a - a_2\lambda_3{}^2\right)/2a_1 a$.

Under the centralized policy (8) the expected social loss becomes:

$$EL^U\left(r^c\right) = Y\sigma^2_{\mu} + \bar{Y}\sigma^2_{\bar{\mu}} \tag{10b}$$

where: $Y = \dfrac{\left(\lambda_1 a_1 - \left(a_1 - a_2\right)\right)}{2\alpha^2\lambda_1 a_1}$ and: $\bar{Y} = \lambda_2 / 2a_1$.

Since: $Y > X$ and: $\bar{Y} \geq \bar{X}$, we can easily verify that: $EL^U\left(r^u\right) < EL^U\left(r^c\right)$. From (10a-b), we obtain the value of the welfare differential $\left(\Delta EL \equiv EL^U\left(r^c\right) - EL^U\left(r^u\right)\right)$:

$$\Delta EL^U = \left(Y - X\right)\sigma^2_{\mu} + \left(\bar{Y} - \bar{X}\right)\sigma^2_{\bar{\mu}} = \frac{\alpha^2\Sigma^2\left[\Sigma^2\sigma^2_{\mu} + \lambda_3{}^2\sigma^2_{\bar{\mu}}\right]}{2a_1\lambda_1\left(a_1 + \alpha^2\Sigma^2\right)} \tag{11}$$

When the Union is heterogeneous, not only asymmetric shocks but also symmetric shocks are not correctly stabilized with a centralized policymaking, as equation (11) clearly

[11] Of course, nationals shocks μ_i are not independent, since they contain a common factor $\left(\mu\right)$. Notice that our way of modeling shocks is simply a question of definition: asymmetric components of shocks are defined in deviation from mean.

shows. Effectively, as *Proposition 1* establishes, a centralized monetary policy is unable to react to asymmetric shocks, but overreacts to symmetric shocks. The (not so much) surprising point that symmetric shocks are not well stabilized with the centralized policymaking comes from the fact that, in a heterogeneous Union, the multipliers of symmetric shocks differ within the area (because the common interest rate reacts to symmetric shocks and the transmission channel of the interest rate to aggregate demand is asymmetric). Thus a centralized monetary policy cannot take account of this heterogeneity of multipliers.

Furthermore, we can obtain from equation (11): $\frac{d\Delta EL^U}{d\Sigma^2} > 0$. Thus, the more heterogeneous the Union is, the highest the relative cost of a centralized policymaking is. This finding holds whatever the nature of shocks (symmetric or asymmetric) is. *Table 2* simulations clearly show that the welfare cost of a centralized monetary policy (relative to a cooperative one: $\Delta EL^U / EL^U$) may be quite high, reaching more than 10% of welfare if the Union is very heterogeneous. *Table 2* simulations also show that the loss of welfare is higher the larger the preference for output relative to inflation stabilization (λ) is.

Table 2 Social loss differential (in %)

	$\Sigma^2 = 0.25$	$\Sigma^2 = 0.5$	$\Sigma^2 = 0.75$	$\Sigma^2 = 1$
$\lambda = 1$	0.65	1.42	2.27	3.17
$\lambda = 3$	3.29	6.19	8.78	11.11
$\lambda = 5$	4.98	9.46	13.51	17.19

For $\alpha = 2$, $a = 1$ and $\sigma_{\bar{\mu}}^2 = \sigma_{\mu}^2$.

3/ Introducing aversion to divergences in the central bank loss function

We are now interested in contractual solution to the issue of centralized monetary policy. Let us suppose that the Union (the "principal") can delegate monetary policy to a common central bank (the "agent") which feels some degree of aversion towards inflation and income divergences in the Union. Such a delegation can be operated by the appointment of divergence-adverse central banker (as in Rogoff, 1985), by the setting of a divergence-target for monetary policy (as in Svensson, 1997) or by establishing an explicit or implicit contract for the common central bank (as in Walsh, 1995), with divergences oriented penalties[12]. We describe such solutions by the fact that, beyond stabilizing average variables in the Union, the central bank attempts to stabilize inflation and income differentials, measured as the cross section standard error of these variables[13]:

$$L^C = \frac{1}{2}\left[\lambda y^2 + \pi^2 + \theta_y \overline{y}^2 + \theta_\pi \overline{\pi}^2\right] \tag{12}$$

where θ_y and θ_π are the coefficients of aversion towards income and inflation divergences, respectively (or, equivalently, the values of the penalties for divergences in the contract for

[12] Penalties can be of financial or "political" (loss of credibility of the central bank, conflicts with Member States of the Union,…) nature. Walsh (2001) discusses in some details different institutional arrangements that corresponds in practice to contracts for central banker, in particular the « *Policy Target Agreement* » established in 1989 in New Zealand.

[13] Defined as: $\overline{y} = \left[\frac{1}{n}\sum_{i=1}^{n}(y_i - y)^2\right]^{1/2}$ et $\overline{\pi} = \left[\frac{1}{n}\sum_{i=1}^{n}(\pi_i - \pi)^2\right]^{1/2}$

the central banker). In this section, we search for optimal values for θ_y and θ_π. The following proposition shows that we can find a simple optimal contract for the common central bank, such that minimizing L^C in (12) amounts to minimizing L^U in (6).

Proposition 2:

If the different Member States of the monetary Union and the common central bank share the same preferences for the stabilization of output and inflation (say, λ and 1 respectively), the first best solution for monetary policy can be obtained by an optimal contract that penalizes the common central bank from inflation and output divergences in the Union. In the optimal contract, the penalties imposed on inflation (respectively on output) divergences correspond to the relative weight of inflation (respectively output) in the social welfare function. Thus, the optimal contract for the common central bank is such as: $\theta_y^* = \lambda$ and $\theta_\pi^* = 1$.

Proof: By minimizing (6) with respect to r, we obtain:

$$\frac{\partial L^U(r)}{\partial r} = \frac{\lambda}{n}\sum_{i=1}^{n} y_i \frac{\partial y_i}{\partial r} + \frac{1}{n}\sum_{i=1}^{n} \pi_i \frac{\partial \pi_i}{\partial r} \tag{13a}$$

By minimizing (12) with respect to r and rearranging, we obtain:

$$\frac{\partial L^C(r)}{\partial r} = y(\lambda-\theta_y)\frac{\partial y}{\partial r} + \pi(1-\theta_\pi)\frac{\partial \pi}{\partial r} + \frac{\theta_y}{n}\sum_{i=1}^{n} y_i \frac{\partial y_i}{\partial r} + \frac{\theta_\pi}{n}\sum_{i=1}^{n} \pi_i \frac{\partial \pi_i}{\partial r} \tag{13b}$$

We can easily observe that expressions (13a) and (13b) are identical if $\theta_y = \lambda$ and $\theta_\pi = 1$, so are loss functions (6) and (12) identical in this case. Thus, under the optimal contract, the centralized monetary regime with aversion to divergences is efficient and leads to the optimal regime[14].

Proposition 2 shows that a simple "optimal contract" for the central bank can enforce the optimal solution. This result is similar to Walsh (1995), except that Walsh deals with inflation bias of monetary policy, while we exclusively deal with a stabilization problem of monetary policy.

The interpretation of the "optimal contract" is straightforward: for monetary policy to take account of Union heterogeneity, one has to force the common central bank to feel some aversion towards inflation and output divergences. If this degree of aversion towards divergences is well defined, as it is the case under the optimal contract, the common monetary policy produces the first best. Notice that penalties on inflation and output differentials look like a "quadratic" contract for central banker, and correspond to changing preferences for the stabilization of divergences relative to the stabilization of Union-wide magnitudes[15].

An illustration

[14] Notice that this result does not depend on a particular form of supply or demand functions: the optimal contract in *Proposition 2* is not model dependent.

[15] One can notice the analogy with the analysis of Rogoff (1985), in which relative preferences for the stabilization of output relative to inflation have to be changed.

Proposition 2 is established for a general case. In our model, minimizing (12) provides the following relation, in place of (8):

$$r = r^c = \psi_1^c \mu^s + \psi_2^c \mu^d + \psi_3^c \sum_{i=1}^{n} \left(\varepsilon_i \bar{\mu}_i^s \right) + \psi_4^c \sum_{i=1}^{n} \left(\varepsilon_i \bar{\mu}_i^d \right) \tag{14}$$

with: $\psi_1^c = -a_1 \Phi$, $\psi_2^c = a_1 \lambda_1 \Phi$, $\psi_3^c = \frac{\alpha^2}{n} (\lambda_3 + \phi_2) \Phi$ and: $\psi_4^c = \frac{\alpha^2}{n} (\lambda_1 + \phi_1) \Phi$, and we use the notation: $\phi_1 = \alpha^2 \left(\theta_y - \lambda \right) + \theta_\pi - 1$, $\phi_2 = \alpha\beta \left(\theta_y - \lambda \right) + 1 - \theta_\pi$ and: $\Phi = \left(a + a_2 \phi_1 \right)^{-1}$.

One can easily verify that (14) corresponds to (9) if $\theta_y^* = \lambda$ and $\theta_\pi^* = 1$, and to (8) if $\theta_y = \theta_\pi = 0$. Reintroducing this rate of interest into equilibrium values of inflation and output, we can express the expected social loss EL^U under the interest rule (14) for any values of θ_y and θ_π [16].

$$EL^U \left(r^c \right) = Z \sigma_\mu^2 + \bar{Z} \sigma_{\bar{\mu}}^2 \tag{15}$$

Let us now compute the differential of welfare associated with a centralized policymaking compared to a cooperative one, namely:

$$\Delta EL = EL^U \left(r^c \right) - EL^U \left(r^u \right) = \left(Z - X \right) \sigma_\mu^2 + \left(\bar{Z} - \bar{X} \right) \sigma_{\bar{\mu}}^2 \tag{16}$$

where: $Z - X = a_1 a_2^{\,2} \left(\phi_1 \right)^2 \Phi^2 / 2a\alpha^2 \geq 0$, $\bar{Z} - \bar{X} = a_2 \left[\lambda_1 \left(\alpha + \beta \right) \phi_2 + \alpha a_2 \phi_3 \right]^2 \Phi^2 / 2a \geq 0$; and we use the notation: $\phi_3 = \left(\theta_y - \lambda \right) - \lambda \left(\theta_\pi - 1 \right)$.

Relation (16) clearly shows that both symmetric and idiosyncratic shocks are not adequately stabilized under the centralized regime, when the Union is heterogeneous. Let us deal with these two questions separately.

Concerning the *symmetric* component of supply shocks, a centralized policymaking reaches the same social loss than the optimal one $\left(X = Z \right)$ if: $\phi_1 = 0$, namely if:

$$\theta_\pi = \theta_\pi^s = 1 - \alpha^2 \left(\theta_y - \lambda \right) \tag{17}$$

This value is also the one that minimizes the social loss function $\left(dEL^U \left(r^c \right) / d\theta_\pi = 0 \right)$ if there is no asymmetric shock $\left(\sigma_{\bar{\mu}}^2 = 0 \right)$.

---

16 On hypothesis i),ii) and iii) above, we can compute: $Z = \left[1 - a_1 \left(a + 2a_2 \phi_1 \right) \Phi^2 \right] / 2\alpha^2$, and: $\bar{Z} = \left\{ \lambda_2 - a_2 \left(\phi_2 + \lambda_3 \right) \left[\left(a + 2a_2 \phi_1 \right) \lambda_3 - a\phi_2 \right] \Phi^2 \right\} / 2a_1$. We can verify that $Z = X$ and $\bar{Z} = \bar{X}$ for $\theta_y = \lambda$ and $\theta_\pi = 1$, and that $Z = Y$ and $\bar{Z} = \bar{Y}$ for $\theta_y = 0$ and $\theta_\pi = 0$, so that expression (15) corresponds respectively to expressions (10a) and (10b) in these cases.

Concerning the asymmetric component of supply shocks, a centralized policymaking reaches the same social loss than the optimal one if: $\overline{X} = \overline{Z}$, namely if:

$$\theta_\pi = \theta_\pi^a = 1 + \alpha^2 \Omega \left(\theta_y - \lambda \right) \tag{18}$$

where: $\Omega = \dfrac{a_2 + \beta(\alpha+\beta)\lambda_1}{\alpha(\alpha+\beta)\lambda_1 + \lambda\alpha^2 a_2}$

This value is also the one that minimizes the social loss function $\left(dEL^U \left(r^c \right) / d\theta_\pi = 0 \right)$ if there is no symmetric shock $\left(\sigma_\mu^2 = 0 \right)$.

We can notice that: $\theta_\pi^s = \theta_\pi^a = 1$ if $\theta_y = \lambda$, finding the optimal contract of *Proposition 2*. But for non-optimal values of the central bank aversion for output divergences (that is $\theta_y \neq \lambda$), there is a conflict between stabilizing symmetric and anti-symmetric components of supply shocks. Effectively, θ_π^s negatively depends on θ_y, while θ_π^a positively depends on it.

This characteristic can be explained as follows. With a centralized monetary policy, the interest rate is too reactive to symmetric supply shocks, as we have seen in *Proposition 1.* Raising the penalty on inflation divergences decreases the response of the interest rate to symmetric shocks and has a stabilizing effect on output differential in the Union (in 4b), thus allowing to a lower value of the penalty on output divergences. Therefore, θ_y and θ_π^s are negatively linked, because both penalties are substitutable instruments in the case of symmetric supply shocks only. On the other hand, the centralized monetary policy implies an insufficient reaction of the interest rate to asymmetric supply shocks. Introducing a penalty on one objective (inflation or output) differential increases the variability of the interest rate, thus raising the differential of the other objective. In this case, obtaining a suitable stabilization of both objectives simultaneously requires θ_y and θ_π^a to move in the same direction: both penalties are complementary instruments in the case of asymmetric supply shocks.

Thus, if $\theta_y < \lambda$, a situation that we favor in section 5, stabilizing symmetric supply shocks would call for a higher than one coefficient of aversion towards inflation divergences $\left(\theta_\pi^s > 1 \right)$, but stabilizing asymmetric supply shocks would require a lower than one coefficient of aversion towards inflation divergences $\left(\theta_\pi^a < 1 \right)$. The reverse is true if $\theta_y > \lambda$. These results are summarized in *Figure 1*[17]:

In *Figure 1*, the decreasing straight line corresponds to the $\left(\theta_y, \theta_\pi \right)$ couples that ensure appropriate stabilization of symmetric shocks[18], and the increasing straight line corresponds to the couples that ensure appropriate stabilization of asymmetric shocks. Optimal contract is obtained at the intersection point of these two lines: optimal penalties $\theta_y^* = \lambda = 1$ and $\theta_\pi^* = 1$ ensure appropriate stabilization of both types of shocks.

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[17] Unless other information, we choose: $\alpha = 2$, $a = 1$, $\lambda = 1$, $\sigma_\mu^2 = \sigma_{\overline{\mu}}^2 = 1$ in all simulations.

[18] This line also corresponds to the appropriate stabilization of demand shocks; symmetric or asymmetric (see *Appendix 2*). Thus, to obtain an optimal stabilization of demand shocks, only one condition is needed, showing that the analysis of supply shocks is more general (see *Appendix 2* for the economic interpretation of this property).

Figure 1 – Best value for θ_π in function of θ_y

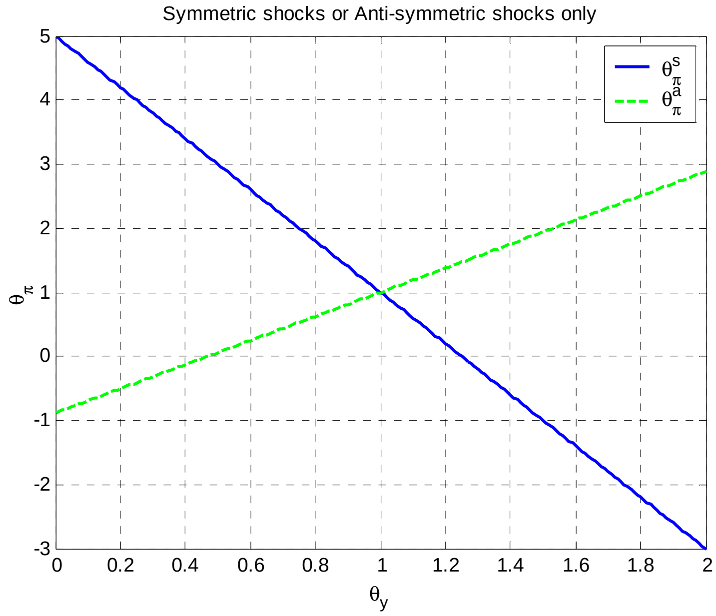


Figure 1 illustrates the conflict between stabilizing symmetric and asymmetric components of supply shocks: to stabilize the symmetric component, θ_y and θ_π must be negatively linked, while they have to be positively linked to stabilize asymmetric shocks. Independently of parameter values, removing the inefficiencies associated to the sub-optimal stabilization of symmetric shocks when $\theta_y = 0$ requires a higher than one value for θ_π ($\theta_\pi^s = 5$ in *Figure 1*), but removing the inefficiencies associated to the sub-optimal stabilization of asymmetric shocks requires a lower than one value for θ_π ($\theta_\pi^a = -0.8$ in *Figure 1*). Section 5 below studies the potential conflict between stabilizing symmetric and asymmetric shocks when the optimal contract cannot be implemented

National losses under the "optimal" contract

A central question about the feasibility of the optimal contract for the common central bank concerns its effects on national welfare in each country of the Union. On this point, it can be seen that the optimal contract does not make things better for all participants of Union. As an illustration, in a two countries (h and f) setting, *Figure 2* simulations[19] show that only one country takes benefits from the implementation of the optimal contract, while the other suffers from it.

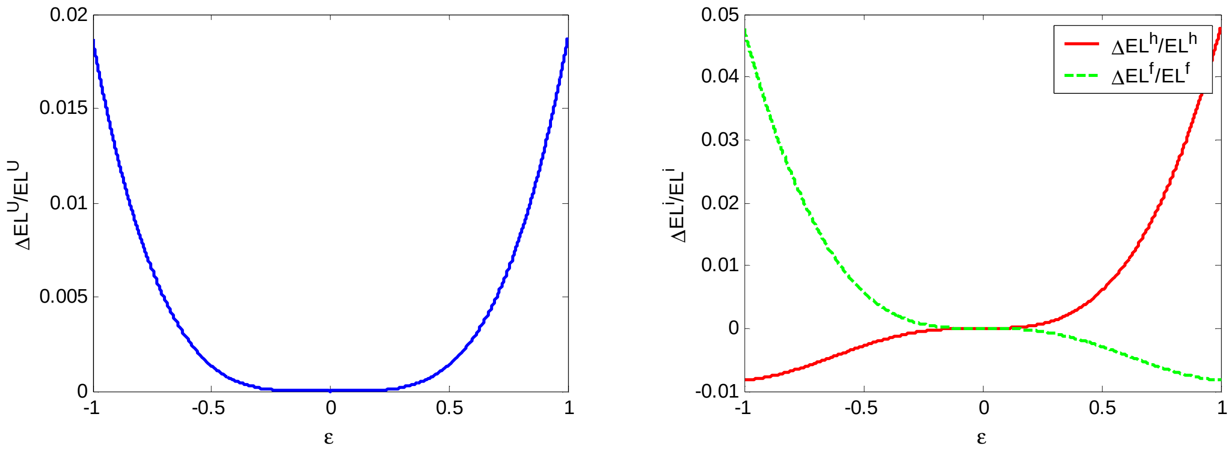
Figure 2 depicts welfare loss differentials for the Union (on the left side) and for the Member States (on the right side) between a centralized monetary policy with $\theta_\pi = \theta_y = 0$ and the optimal (from the Union-wide welfare point of view) policy with $\theta_\pi = 1$ and $\theta_y = \lambda$, for different values of the degree of heterogeneity in Union (ε). This *Figure* confirms the fact that only one country (country h if $\varepsilon > 0$ and country f if $\varepsilon < 0$), takes benefits from the

[19] In *Figure* 2 we set: $n = 2$. Notice that in a two-country model: $\varepsilon_1 = \varepsilon$ and $\varepsilon_2 = -\varepsilon$.

implementation of the optimal contract, and that these benefits (or these losses) are higher the larger the heterogeneity of Union is.

Therefore, modifying common central bank preferences, even to implement the optimal contract, would be a source of potential conflicts between Member States of the Union, especially in a heterogeneous Union.

Figure 2 – Union-wide and Member-States social loss differentials (in %)



4/ The optimal contract with independent central bank preferences for output and inflation stabilization

One important limit about *Proposition 2* is that the different Member States of the Union and the common central bank must share the same preferences. In the opposite case, it is still possible to find an optimal contract which removes the inefficiency associated with a centralized monetary policy, but this contract is more complicated and the penalties towards inflation and output divergences become model-dependent.

Let us suppose now that the common central bank possess own preferences for the stabilization of output relative to inflation, namely: $\tilde{\lambda} \neq \lambda$. The objective of the central bank becomes:

$$L^c = \frac{1}{2}\left[\tilde{\lambda} y^2 + \pi^2 + \theta_y \bar{y}^2 + \theta_\pi \bar{\pi}^2\right] \tag{12'}$$

We suppose, as usual, that the central bank is more concerned with inflation stabilization than society $\left(\tilde{\lambda} \leq \lambda\right)$. The interest rule that comes from the minimization of (12') is analogous to equation (14) above[20]:

$$r = r^c = \tilde{\psi}_1^c \mu^s + \tilde{\psi}_2^c \mu^d + \tilde{\psi}_3^c \sum_{i=1}^{n}\left(\varepsilon_i \bar{\mu}_i^s\right) + \tilde{\psi}_4^c \sum_{i=1}^{n}\left(\varepsilon_i \bar{\mu}_i^d\right) \tag{14'}$$

[20] We use trivial changes in notations: $\tilde{\psi}_1^c = -a_1\tilde{\Phi}$, $\tilde{\psi}_2^c = a_1\tilde{\lambda}_1\tilde{\Phi}$, $\tilde{\psi}_3^c = \frac{\alpha^2}{n}\left(\tilde{\lambda}_3 + \tilde{\phi}_2\right)\tilde{\Phi}$ and $\tilde{\psi}_4^c = \frac{\alpha^2}{n}\left(\tilde{\lambda}_1 + \tilde{\phi}_1\right)\tilde{\Phi}$, with: $\tilde{\phi}_1 = \alpha^2\left(\theta_y - \tilde{\lambda}\right) + \theta_\pi - 1$, $\tilde{\phi}_2 = \alpha\beta\left(\theta_y - \tilde{\lambda}\right) + 1 - \theta_\pi$, $\tilde{\Phi} = \frac{1}{\tilde{a} + a_2\tilde{\phi}_1}$, and $\tilde{a} = \tilde{\lambda}_1 a / \lambda_1$. $\tilde{\lambda}_3 = \alpha\beta\tilde{\lambda} - 1$, $\tilde{\lambda}_1 = 1 + \alpha^2\tilde{\lambda}$, $\tilde{\lambda}_2 = 1 + \beta^2\tilde{\lambda}$

Proposition 3:

Suppose that there are only supply shocks $\left(\sigma^2_{\bar{\mu}_i^d} = \sigma^2_{\mu^d} = 0, \forall i\right)$. *If the common central bank is more concerned with the stabilization of inflation relative to output, compared with social preferences in the Union, namely if* $\tilde{\lambda} \le \lambda$, *the first best solution for monetary policy can be obtained by an optimal contract that penalizes the common central bank from inflation and output divergences in the Union. In the optimal contract, the penalties imposed on inflation (respectively on output) divergences are higher than the relative weighs of inflation (respectively output) in the social welfare function. Thus, the optimal contract for the common central bank is such as:* $\theta_y^* \ge \lambda$ *and* $\theta_\pi^* \ge 1$.

Proof: The social loss function associated to the interest rule (14') is now[21]:

$$EL^U\left(r^c\right) = \tilde{Z}\sigma^2_\mu + \tilde{\tilde{Z}}\sigma^2_{\bar{\mu}} \qquad (15')$$

And the differential of welfare associated with a centralized policymaking compared to a cooperative one is:

$$\Delta EL = EL^U\left(r^c\right) - EL^U\left(r^u\right) = \left(\tilde{Z} - X\right)\sigma^2_\mu + \left(\tilde{\tilde{Z}} - \bar{X}\right)\sigma^2_{\bar{\mu}} \qquad (16')$$

where:

$$\tilde{Z} - X = \frac{a_1}{2\alpha^2 a}\left[a_2\phi_1 - \alpha^2 a_1\left(\lambda - \tilde{\lambda}\right)\right]^2 \tilde{\Phi}^2 \ge 0$$

$$\tilde{\tilde{Z}} - \bar{X} = \frac{a_2}{2a}\left[\left(\alpha + \beta\right)\left[\lambda_1\phi_2 + \alpha^2\lambda_3\left(\lambda - \tilde{\lambda}\right)\right] + \alpha a_2\phi_3\right]^2 \tilde{\Phi}^2 \ge 0$$

If $\tilde{\lambda} = \lambda = \theta_y$ and $\theta_\pi = 1$, the differential of welfare is zero. But if $\tilde{\lambda} \le \lambda$, the differential of welfare is positive even if $\theta_y = \tilde{\lambda}$ and $\theta_\pi = 1$[22]. Consequently, the optimal contract for monetary policy is not $\theta_y = \tilde{\lambda}$ and $\theta_\pi = 1$.

Concerning the symmetric component of supply shocks (*i.e.* if $\sigma^2_{\bar{\mu}} = 0$), a centralized regime with aversion toward divergences produces the same social loss as the optimal "cooperative" regime if:

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[21] where: $\tilde{Z} = \left\{1 - a_1\left[a + 2\left(a_2\phi_1 - a_1\alpha^2\left(\lambda - \tilde{\lambda}\right)\right)\right]\tilde{\Phi}^2\right\} / 2\alpha^2$ and $\tilde{\tilde{Z}} = \left\{\lambda_2 - a_2\left(\phi_2 + \lambda_3\right)\left[\lambda_3\left(2a_2\tilde{\phi}_1 + \tilde{a} - \alpha^2\left(a_1 + a_2\right)\left(\lambda - \tilde{\lambda}\right)\right) - a\phi_2\right]\tilde{\Phi}^2\right\} / 2a_1$. We can easily verify that: $\tilde{Z} = Z$, $\tilde{\tilde{Z}} = \bar{Z}$ if $\lambda = \tilde{\lambda}$.

[22] Effectively, for $\theta_y = \tilde{\lambda}$ and $\theta_\pi = 1$, we have: $\tilde{Z} - X = \dfrac{a_1\alpha^2\left(\lambda - \tilde{\lambda}\right)^2}{2a\tilde{\lambda}_1^2} \ge 0$ and $\tilde{\tilde{Z}} - \bar{X} = \dfrac{a_2\alpha^2\left(\lambda - \tilde{\lambda}\right)^2}{2a\tilde{\lambda}_1^2} \ge 0$.

$$\theta_\pi = \theta_\pi^s = 1 - \alpha^2\left(\theta_y - \lambda\right) + \alpha^2 \frac{a_1}{a_2}\left(\lambda - \tilde{\lambda}\right) \tag{17'}$$

Concerning the asymmetric component of supply shocks (*i.e.* if $\sigma_\mu^2 = 0$), we obtain the same social loss under both regimes if:

$$\theta_\pi = \theta_\pi^a = 1 + \alpha^2\left[\Omega\left(\theta_y - \lambda\right) + \frac{\alpha(\alpha+\beta)\lambda_3\left(\lambda - \tilde{\lambda}\right)}{\alpha(\alpha+\beta)\lambda_1 + \alpha^2 a_2 \lambda}\right] \tag{18'}$$

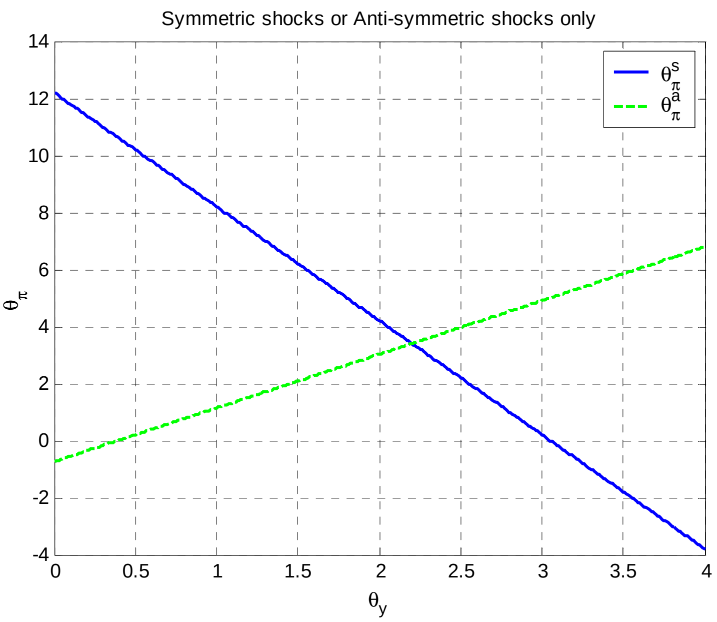
From (17')-(18'), we easily obtain the values of the coefficients of aversion towards output and inflation divergences under the optimal contract, namely:

$$\begin{aligned} \theta_y^* &= \lambda + \frac{(\alpha+\beta)}{\alpha\Sigma^2}\left(\lambda - \tilde{\lambda}\right) \geq \lambda \\ \theta_\pi^* &= 1 + \left[\frac{\beta(\alpha+\beta)}{\Sigma^2}\right]\left(\lambda - \tilde{\lambda}\right) \geq 1 \end{aligned} \tag{19}$$

This proves *Proposition 3*.

These results are depicted in *Figure 3* for $\tilde{\lambda} = 0.8 < \lambda = 1$. θ_π^s and θ_π^a are still respectively decreasing and increasing function in θ_y, but, when $\tilde{\lambda} < \lambda$, the two curves moves upwards compared with *Figure 1*. As a result, the optimal values of θ_π^s and θ_π^a are higher than they was when $\tilde{\lambda} = \lambda$. In *Figure 3* special case: $\theta_y^* = 2.2$ and $\theta_\pi^* = 3.4$, instead of $\theta_\pi^s = \theta_\pi^a = 1$.

Figure 3 – Best value for θ_π *in function of* θ_y $\left(\tilde{\lambda} = 0.8\right)$



The interpretation of *Proposition 3* is the following. With independent preferences of the central bank $\left(\tilde{\lambda} \neq \lambda\right)$ and centralized policymaking, monetary policy is affected by two biases. Sufficiently high values of penalties allow removing the bias associated to the centralized policymaking *and* the bias associated to independent preference for the stabilization of output relative to inflation.

Remark that in relations (19), the optimal values of aversion towards inflation and output divergences are decreasing function of the degree of heterogeneity in the Union $\left(\Sigma^2\right)$. The more heterogeneous the Union is, the lesser the common central bank should worry about inflation and output divergences under the optimal contract.

Let us examine more closely this apparent paradox. If the Union was homogenous, no optimal contract could be reached, because no finite value of penalties could remove the bias associated to independent preferences[23]. Effectively, in a homogenous Union $\left(\Sigma^2 = 0\right)$, minimizing L^U in (6) would correspond to minimizing $L = \frac{1}{2}\left[\lambda y^2 + \pi^2\right]$, as we have seen, and no penalty on divergences would allow equalizing this quantity to $L^c = \frac{1}{2}\left[\tilde{\lambda} y^2 + \pi^2\right]$. Thus, no value of θ_π or θ_y would totally remove the inefficiencies associated with the "wrong" central bank loss function, as show the values of penalties in equation (19) which tend to infinity.

In a heterogeneous Union, on the contrary, imposing penalties on inflation and output divergences allows totally removing the inefficiencies associated with the "wrong" central bank loss function. Sufficiently "high" values of the penalties in equation (19) correct both the bias associated with a centralized policymaking *and* the bias associated with the particular preference for output/inflation stabilization. In the optimal contract, penalties have to be higher than if $\tilde{\lambda} = \lambda$ (namely, 1), but the more heterogeneous the Union is, the lesser these penalties have to be. Thus, the gap between optimal penalties when $\tilde{\lambda} = \lambda$ and their values when $\tilde{\lambda} < \lambda$ negatively depends on the degree of Union heterogeneity, because heterogeneity gives an instrument for correcting the bias associated with the "wrong" preference parameter $\tilde{\lambda}$. Taking the logic to extreme, in a strongly heterogeneous Union $\left(\Sigma^2 \to +\infty\right)$, the relative preference for output/inflation stabilization is irrelevant, because the optimal policy is: $r^c = r^U = 0$ in (9) and (14'), and the optimal contract is the same as in section 3.

5/ Second-best contracts for monetary policy

However, the optimal contract does not seem to characterize the behavior of the European Central Bank. Effectively, in the Euro area, the recent monetary policy debate has tended to focus on the cost of a monetary policy uniquely designed on the base of Union-wide quantities and on the difficulties for defining an a suitable common monetary policy in the presence of large inflation differentials, with few references to income divergences. Moreover, it seems difficult to design monetary policy in function of output-gap or growth differentials in EMU, since these differentials reflect structural adjustment and catching up of

[23] Simply because the common interest rate cannot affect cross-countries standard-error of inflation and output if the Union is homogenous. Therefore, penalties on divergences cannot change the response of "centralized" interest rate $\left(r^c\right)$ to shocks, and the equality $r^c = r^u$ never can be reached in an homogenous Union with $\tilde{\lambda} \neq \lambda$. Thus, optimal penalties in (19) go to infinity if $\Sigma^2 \to 0$.

less developed Member States, and are outside the province of current interest rate policy. Even if inflation divergences also possess a structural component, they directly affect the central bank ability of defining a "good" inflation rate for the area, and the European Central Bank does probably keep more watch on inflation differentials than on output differentials.

In what follows, we wonder about the existence of a "*second best*" contract, when the common central bank shares the social relative preferences for output and inflation stabilization $\left(\tilde{\lambda}=\lambda\right)$, as in sections 2 and 3, and worries about inflation differentials, but without being endowed with the optimal degree of aversion towards output divergences $\left(\theta_y \neq \theta_y^* = \lambda\right)$. In other words, we search for the optimal degree of aversion towards inflation divergences in function of the degree of aversion towards output divergences (possibly zero).

Relations (17) and (18) and *Figure 1* above already exhibit the best value of θ_π in function of θ_y, in two special cases: with symmetric shocks only ($\sigma_{\bar{\mu}}^2 = 0$ for 17) or with asymmetric shocks only ($\sigma_{\bar{\mu}}^2 = 0$ for 18). Now, we search for the best reaction function $\theta_\pi = f\left(\theta_y\right)$ in the general case with both symmetric and asymmetric shocks. Notice that the slope of this reaction function depends on the relative sizes of symmetric and asymmetric shocks, since the reaction function is decreasing in the presence of symmetric shocks only (17) and increasing if there are asymmetric shocks only (18). Let us denote by $\sigma = \sigma_{\bar{\mu}}^2 / \sigma_\mu^2$ the ratio of variances of asymmetric to symmetric shocks (*i.e.* the relative "size" of asymmetric to symmetric shocks). With both symmetric and asymmetric shocks, the degree of aversion towards inflation divergences $\left(\theta_\pi^*\right)$ that minimizes the welfare differential in function of the coefficient of aversion for output divergences $\left(\theta_y\right)$ can be expressed as[24]:

$$\theta_\pi^* - 1 = \Theta\left(\theta_y - \lambda\right) \tag{20}$$

Coefficient Θ depends on the variance of symmetric and asymmetric shocks (σ), but for admissible parameter values, asymmetric shocks dominate even if their variance is very small compared to the variance of symmetric shocks. Thus, the relation between θ_y and θ_π^* is most probably positive[25].

Furthermore, when the common central bank is not concerned by output divergences $\left(\theta_y = 0\right)$, solving the problem of asymmetric shocks generally requires a *negative* optimal degree of aversion towards inflation divergences. *Figure 4*, which represents θ_π^* as a function of θ_y for different values of the ratio of variances $\sigma = \sigma_{\bar{\mu}}^2 / \sigma_\mu^2$, depicts this feature. Effectively, for our simulation values, the optimal "second best" penalty for inflation divergences when the central bank do not carry about output divergences $\left(\theta_y = 0\right)$ becomes negative as soon as the relative "size" of asymmetric to symmetric shocks is higher than $\sigma \geq 0.05$. In this case, from

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[24] In this expression: $\Theta = \dfrac{-a_1 a_2 + \alpha\left(a_2 + (\alpha+\beta)\beta\lambda_1\right)\left[(\alpha+\beta)\lambda_1 + \alpha a_2 \theta_y\right]\sigma}{a_1\left(\Sigma^2 + \lambda_1^{\ 2}\sigma\right) + a_2\alpha\left[(\alpha+\beta)\lambda\lambda_1 + \theta_y\left((\alpha+\beta)\lambda_1 + \alpha a_2\lambda\right)\right]\sigma}$. Relation (20) is obtained by minimizing (15) or (16) with respect to θ_π, θ_y given.

[25] For above mentioned simulation values, relation (20) is positively sloped as soon as $\sigma \geq 0.02$.

the Union-wide welfare perspective, a common central bank that worries about inflation divergences without taking care of output differentials would not be a good idea.

Figure 4 – Best value for θ_π in function of θ_y

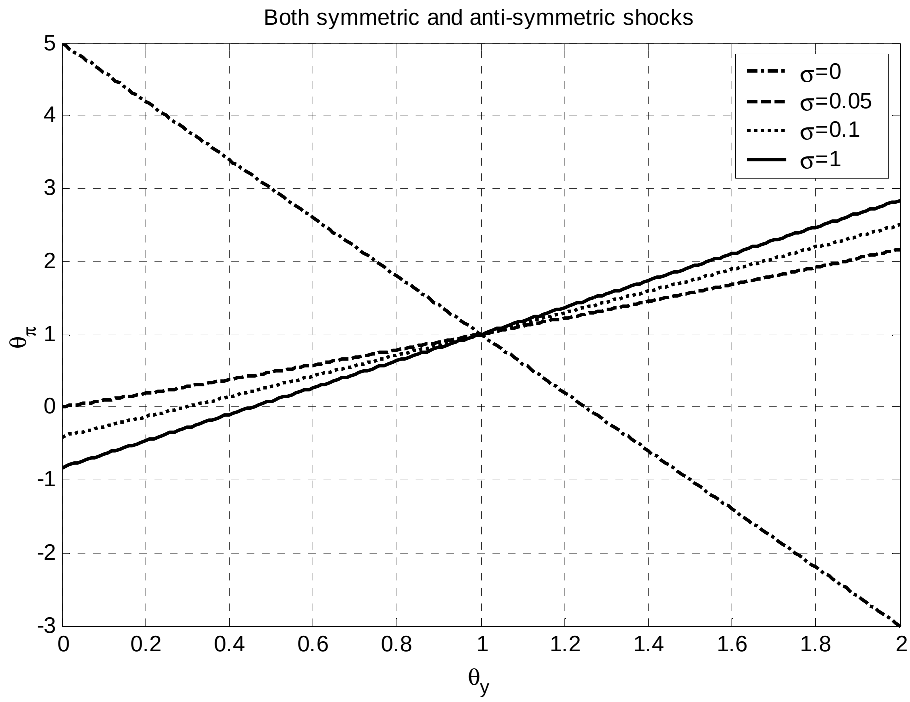
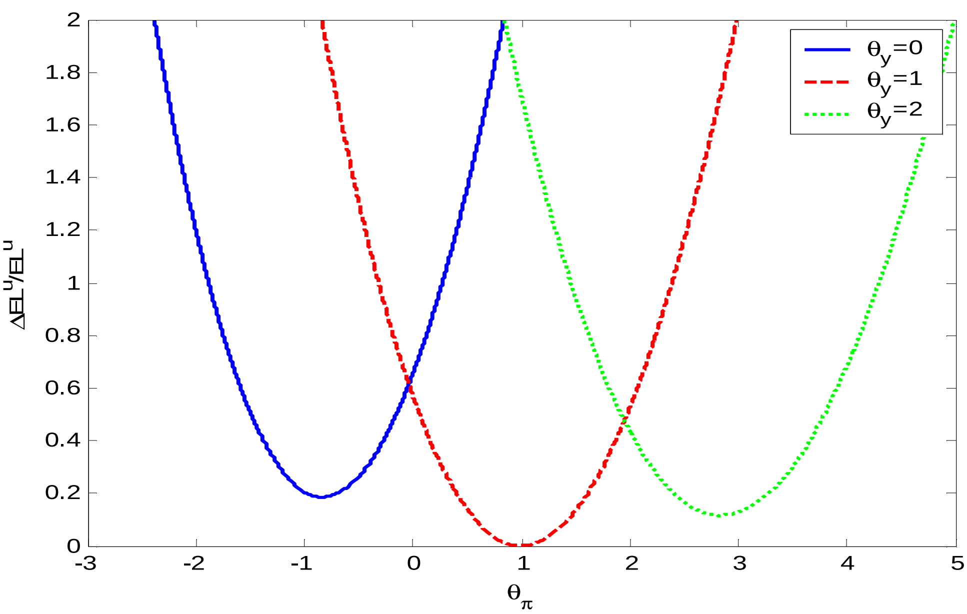


Figure 5 – Social-loss differential as a function of θ_π (in %)



In *Figure 5*, we depict the differential of welfare (16) with both symmetric and asymmetric shocks. As we have seen, this differential is fully removed only on the optimal contract (here: $\theta_y = \theta_\pi = 1$). If the degree of aversion towards output divergences is set sub-optimally $(\theta_y \neq 1)$, on the contrary, the second best value for the coefficient for inflation differentials conducts to a net loss of welfare compared to the "cooperative" monetary policy.

Notice in particular that, if the central bank is not concerned about output differentials $\left(\theta_y = 0\right)$, the best coefficient for inflation divergences it not necessarily positive. On the contrary, if the common central bank if "too much" affected by output divergences ($\theta_y = 2$, for example), the second best penalty towards inflation divergences becomes larger than one.

Finally, our model shows that a common central bank which focuses on inflation differential, and disregards output differential may be detrimental to the Union welfare, especially if the Union is stricken by large asymmetric shocks. In such a situation, penalties on inflation and output differentials are complementary. On the contrary, if symmetric shocks are pre-eminent, a contract that penalizes the central bank from inflation divergences only may be beneficial to the Union welfare. In any case, the analysis shows the importance of taking account of the precise nature of shocks in assessing the welfare gains associated to different institutional arrangements.

6/ Conclusion

Should a common central bank in a heterogeneous monetary Union worry about inflation and output differentials? In this paper, we have shown at a theoretical level that an optimal contract for the common central bank can be found, from the point of view of Union-wide welfare. This contract penalizes the central bank from inflation and output divergences in Union. On the other hand, penalizing the central bank from inflation divergences only is not necessarily a better solution than a "centralized" policymaking that is only concerned with Union-wide magnitudes. Furthermore, even the optimal contract is difficult to implement, because only some Member States takes advantage of this contract, while it is detrimental to the welfare of others. Modifying common central bank preferences is therefore a source of potential conflicts between Member States of the Union.

Of course, the optimal contract found in this paper is subject to usual criticisms directed at contractual literature in monetary policy. However, the fiction of a contract for central banker simply describes the fact that the central bank must be forced, implicitly or explicitly, to watch closely to divergences in the Union. The form of the optimal contract is not much complicated than the optimal contract derived by Walsh (1995) to solve a credibility problem of monetary policy, which results in a linear penalty on inflation. The difference is that in our model, monetary policy is not subject to a time inconsistency problem, but to a stabilization problem, which results in linear penalties on inflation and output divergences.

In the Euro area, such divergences come not only from idiosyncratic shocks but also from structural asymmetries between Member States. Even if our model does not completely takes account of these asymmetries, which may reflect different level of economic development during the catching up process, introducing structural heterogeneity into the transmission channel of monetary policy may provide a first step towards a more general framework.

Such a more general framework should extend the present analysis to an open monetary Union, since it would be interesting to see how penalties on national divergences are affected by exchanges with foreign countries. Studying more explicitly the different channels of heterogeneity in the Union could also produce interesting results about the form of the contract for the common central banker. Finally, another interesting extension of the present model would be to investigate how the optimal contract for the central bank in a heterogeneous Union is affected by the behaviour of national governments, in a framework where governments minimize their own loss function. Such extensions are unlikely to modify the optimal contract for the central bank, which is not model dependent, have we have seen, but probably strongly affect the contract in a second best world.

Appendix 1: Resolution of the model

From the first order condition for the minimization of (6), namely: $\frac{dL^U}{dr}=0$, we obtain the optimal interest rate:

$$r^u = \frac{\sum_{i=1}^{n}\left[(\beta b+\alpha b_i)\left[(\alpha+\beta)\lambda_1\mu^d-(\alpha+\beta)\mu^s+\alpha\lambda_1\overline{\mu}_i^d+\alpha\lambda_3\overline{\mu}_i^s\right]\right]}{\sum_{i=1}^{n}\left[(\beta b+\alpha b_i)^2\lambda_1\right]}$$

Knowing that: $\sum_{i=1}^{n}\varepsilon_i=0$, we can compute: $\sum_{i=1}^{n}b_i=nb$, $\sum_{i=1}^{n}\left(b_i^2\right)=nb^2\left(1+\Sigma^2\right)$, $\sum_{i=1}^{n}\left(b_i\overline{\mu}_i^s\right)=b\sum_{i=1}^{n}\left(\varepsilon_i\overline{\mu}_i^s\right)$, and: $\sum_{i=1}^{n}\left(b_i\overline{\mu}_i^d\right)=b\sum_{i=1}^{n}\left(\varepsilon_i\overline{\mu}_i^d\right)$. Thus, optimal interest rate becomes:

$$br^u = \frac{\frac{(\alpha+\beta)^2}{\lambda_1}\mu^s+(\alpha+\beta)^2\mu^d+\frac{\alpha^2\lambda_3}{n\lambda_1}\sum_{i=1}^{n}\left(\varepsilon_i\overline{\mu}_i^s\right)+\frac{\alpha^2}{n}\sum_{i=1}^{n}\left(\varepsilon_i\overline{\mu}_i^d\right)}{(\alpha+\beta)^2+\alpha^2\Sigma^2}$$

Since the effect of interest rate passes trough b coefficient in demand functions, equilibrium solutions are independent of b, and we can normalize $b=1$ (the same reasoning applies for the centralized regime). Setting: $a_1=(\alpha+\beta)^2$, $a_2=\alpha^2\Sigma^2$ and $a=\lambda_1(a_1+a_2)$, we find equation (9) of main text.

From the first order condition for the minimization of the common central bank loss function with aversion towards divergences (12), namely: $\frac{dL^C}{dr}=0$, we obtain the centralized interest rate, in the general case (for any values of the penalties):

$$r^c = \frac{\frac{b\lambda_1\mu^d-b\mu^s}{\alpha^2}+\frac{1}{n(\alpha+\beta)^2}\sum_{i=1}^{n}(b-b_i)\left[\theta_\pi\left(\overline{\mu}_i^s-\overline{\mu}_i^d\right)-\theta_y\alpha\left(\alpha\overline{\mu}_i^d+\beta\overline{\mu}_i^s\right)\right]}{\frac{b^2\lambda_1}{\alpha^2}+\frac{\left(\theta_\pi+\theta_y\alpha^2\right)}{n(\alpha+\beta)^2}\sum_{i=1}^{n}(b-b_i)^2}$$

Since: $\sum_{i=1}^{n}(b-b_i)=0$, $\sum_{i=1}^{n}(b-b_i)^2=nb^2\Sigma^2$, $\sum_{i=1}^{n}(b-b_i)\overline{\mu}_i^s=-b\sum_{i=1}^{n}\left(\varepsilon_i\overline{\mu}_i^s\right)$, and : $\sum_{i=1}^{n}(b-b_i)\overline{\mu}_i^d=-b\sum_{i=1}^{n}\left(\varepsilon_i\overline{\mu}_i^d\right)$, we have:

$$br^c = \frac{\dfrac{\lambda_1 \mu^d - \mu^s}{\alpha^2} + \dfrac{1}{(\alpha+\beta)^2 n}\left[\left(\theta_\pi + \theta_y \alpha^2\right)\sum_{i=1}^{n}\left(\varepsilon_i \overline{\mu}_i^d\right) + \left(\alpha\lambda\theta_y - \theta_\pi\right)\sum_{i=1}^{n}\left(\varepsilon_i \overline{\mu}_i^s\right)\right]}{\dfrac{\lambda_1}{\alpha^2} + \dfrac{1}{(\alpha+\beta)^2}\left(\theta_\pi + \theta_y \alpha^2\right)\Sigma^2}$$

Setting: $b = 1$, $a_1 = (\alpha+\beta)^2$, $a_2 = \alpha^2\Sigma^2$, $a = \lambda_1(a_1 + a_2)$, $\phi_1 = \alpha^2(\theta_y - \lambda) + \theta_\pi - 1$ and: $\phi_2 = \alpha\beta(\theta_y - \lambda) - (\theta_\pi - 1)$, we find equation (14) of the main text, corresponding to the minimization of (12):

$$r^c = -\frac{a_1}{a + a_2\phi_1}\mu^s + \frac{a_1\lambda_1}{a + a_2\phi_1}\mu^d + \frac{\alpha^2(\lambda_3 + \phi_2)}{n(a + a_2\phi_1)}\sum_{i=1}^{n}\left(\varepsilon_i \overline{\mu}_i^s\right) + \frac{\alpha^2(\lambda_1 + \phi_1)}{n(a + a_2\phi_1)}\sum_{i=1}^{n}\left(\varepsilon_i \overline{\mu}_i^d\right)$$

If: $\theta_y = \theta_\pi = 0$, thus: $\lambda_3 + \phi_2 = 0$ and: $\lambda_1 + \phi_1 = 0$, and we find relation (8) in the main text.

Social loss function in the centralized regime

Suppose first that there is no demand shock $\left(\mu^d = \overline{\mu}_i^d = 0\right)$. Replacing the expression of the interest rate in equilibrium inflation and output (equations 3a, 3b, 4a and 4b of the main text), we find, for the centralized regime:

$$\pi_i^c = \frac{q_i - \lambda_1(\alpha+\beta)}{\alpha\lambda_1(\alpha+\beta)}\mu^s - \frac{1}{\alpha+\beta}\overline{\mu}_i^s$$

$$y_i^c = \frac{q_i}{\alpha\lambda_1(\alpha+\beta)}\mu^s + \frac{\beta}{\alpha+\beta}\overline{\mu}_i^s, \text{ with: } q_i = \beta + \alpha(1+\varepsilon_i)$$

Denoting by: $\sigma_{\overline{\mu}}^2 = \frac{1}{n}\sum_{i=1}^{n}\sigma_{\overline{\mu}_i}^2$ the variance of asymmetric supply shocks, we get the expression of the *ex ante* social loss function:

$$EL^U(r^c) = \frac{1}{n}\sum_{i=1}^{n} EL^i(r^c) = \frac{1}{2\alpha^2}\left[1 - \frac{2(\alpha+\beta)\sum_{i=1}^{n} q_i - \sum_{i=1}^{n} q_i^2}{na_1\lambda_1}\right]\sigma_\mu^2 + \frac{\lambda_2}{2a_1}\sigma_{\overline{\mu}}^2$$

Since: $\sum_{i=1}^{n} q_i = n(\alpha+\beta)$ and: $\sum_{i=1}^{n} q_i^2 = n(a_1 + a_2)$, (A1) corresponds to equation (10b) in the main text:

$$EL^U(r^c) = \left(\frac{\lambda_1 a_1 - (a_1 - a_2)}{2\alpha^2\lambda_1 a_1}\right)\sigma_\mu^2 + \left(\frac{\lambda_2}{2a_1}\right)\sigma_{\overline{\mu}}^2 \qquad \text{(A1)}$$

Social loss function in the optimal regime

For the optimal ('cooperative') regime, we obtain, from the interest rate rule (9):

$$\pi_i^u = \frac{q_i \frac{a_1}{a} - (\alpha+\beta)}{\alpha(\alpha+\beta)} \mu^s - \frac{\alpha \overline{\mu}_i^s + r_i}{\alpha(\alpha+\beta)}$$

$$y_i^u = \frac{a_1 q_i}{\alpha+\beta} \mu^s + \frac{\beta \overline{\mu}_i^s - r_i}{\alpha+\beta}, \text{ with: } r_i = q_i \frac{\alpha^2 \lambda_3}{na} \sum_{i=1}^{n} \left(\varepsilon_i \overline{\mu}_i^s\right).$$

Thus, the expression of the *ex ante* social loss function:

$$EL^U\left(r^u\right) = \frac{1}{n}\sum_{i=1}^{n} EL^i\left(r^u\right) = \frac{1}{2\alpha^2}\left[1-\frac{a_1}{a}\right]\sigma_\mu^2 + \frac{\lambda_2}{2a_1}\sigma_{\overline{\mu}}^2 + \frac{1}{2na_1\alpha^2 a^2} E\left[a^2\lambda_1 \sum_{i=1}^{n} r_i^2 - 2a^2\alpha\lambda_3 \sum_{i=1}^{n} r_i \overline{\mu}_i^s\right]$$

Using the following results: $E\left[\sum_{i=1}^{n} r_i^2\right] = \frac{\left(a_1+a_2\right)\left(\lambda_3\right)^2 \alpha^4}{a^2 n} E\left[\left(\sum_{i=1}^{n}\left(\varepsilon_i \overline{\mu}_i^s\right)\right)^2\right]$ and:

$E\left[\sum_{i=1}^{n} r_i \overline{\mu}_i^s\right] = \frac{\alpha^3 \lambda_3}{an} E\left[\left(\sum_{i=1}^{n}\left(\varepsilon_i \overline{\mu}_i^s\right)\right)^2\right]$, we obtain:

$$EL^U(r^u) = \frac{1}{2\alpha^2}\left[1-\frac{a_1}{a}\right]\sigma_\mu^2 + \frac{\lambda_2}{2a_1}\sigma_{\overline{\mu}}^2 - \frac{\alpha^2\left(\lambda_3\right)^2}{2a_1 a} E\left[\left(\frac{1}{n}\sum_{i=1}^{n}\left(\varepsilon_i \overline{\mu}_i^s\right)\right)^2\right] \qquad \text{(A2)}$$

in the special case of i) independently distributed asymmetric supply shocks $\left(\sigma_{\overline{\mu}_i^s}\sigma_{\overline{\mu}_j^s} = 0\right)$ and ii) same variance of supply shocks in all countries ($\sigma_{\overline{\mu}_i^s}^2 = \sigma_{\overline{\mu}}^2 \ \forall i$), we find relation (10a) in main text, since: $E\left[\left(\frac{1}{n}\sum_{i=1}^{n}\left(\varepsilon_i \overline{\mu}_i^s\right)\right)^2\right] = \frac{\sigma_{\overline{\mu}}^2}{n}\sum_{i=1}^{n}\left(\varepsilon_i^2\right) = \Sigma^2 \sigma_{\overline{\mu}}^2$. As we shall see in this *Appendix*, all results in the main text hold independently of these two assumptions.

Social loss function in the centralized regime with aversion towards divergences $\left(\tilde{\lambda} = \lambda\right)$

With the interest rate definition (14), equilibrium inflation and output are:

$$\pi_i^c = \frac{q_i a_1 \Phi - (\alpha+\beta)}{\alpha(\alpha+\beta)} \mu^s - \frac{\alpha \overline{\mu}_i^s + s_i}{\alpha(\alpha+\beta)}$$

$$y_i^c = \frac{a_1 \Phi q_i}{\alpha+\beta} \mu^s + \frac{\beta \overline{\mu}_i^s - s_i}{\alpha+\beta}, \text{ with: } s_i = q_i \frac{\alpha^2\left(\lambda_3 + \phi_2\right)\Phi}{n} \sum_{i=1}^{n}\left(\varepsilon_i \overline{\mu}_i^s\right)$$

The *ex ante* social loss function is:

$$EL^U\left(r^c\right)=\frac{1}{2\alpha^2}\left[1-a_1\left(a+2a_2\phi_1\right)\Phi^2\right]\sigma_\mu^2+\frac{\lambda_2}{2a_1}\sigma_{\bar{\mu}}^2+\frac{1}{2n\alpha^2a_1}E\left[\lambda_1\sum_{i=1}^n s_i^2-2\alpha\lambda_3\sum_{i=1}^n s_i\bar{\mu}_i^s\right]$$

Using the following results: $E\left[\sum_{i=1}^n s_i^2\right]=\frac{\left(a_1+a_2\right)\alpha^4\Phi^2\left(\lambda_3+\phi_2\right)^2}{n}E\left[\left(\sum_{i=1}^n\left(\varepsilon_i\bar{\mu}_i^s\right)\right)^2\right]$

and $E\left[\sum_{i=1}^n s_i\bar{\mu}_i^s\right]=\frac{\alpha^3\Phi\left(\lambda_3+\phi_2\right)}{n}E\left[\left(\sum_{i=1}^n\left(\varepsilon_i\bar{\mu}_i^s\right)\right)^2\right]$, we find:

$$EL^U\left(r^c\right)=\frac{Z}{2\alpha^2}\sigma_\mu^2+\frac{\lambda_2}{2a_1}\sigma_{\bar{\mu}}^2-\left[\left(a+2a_2\phi_1\right)\lambda_3-a\phi_2\right]\frac{\alpha^2\left(\lambda_3+\phi_2\right)\Phi^2}{2a_1}E\left[\left(\frac{1}{n}\sum_{i=1}^n\left(\varepsilon_i\bar{\mu}_i^s\right)\right)^2\right] \quad \text{(A3)}$$

On the two above mentioned assumptions, one can easily find expression (15) in the main text.

Social loss function in the centralized regime with aversion towards divergences $\left(\tilde{\lambda}\neq\lambda\right)$

With the interest rate definition (14'), equilibrium inflation and output are:

$$\pi_i^c=\frac{q_ia_1\tilde{\Phi}-\left(\alpha+\beta\right)}{\alpha\left(\alpha+\beta\right)}\mu^s-\frac{\alpha\bar{\mu}_i^s+\tilde{s}_i}{\alpha\left(\alpha+\beta\right)}$$

$$y_i^c=\frac{a_1\tilde{\Phi}q_i}{\alpha+\beta}\mu^s+\frac{\beta\bar{\mu}_i^s-\tilde{s}_i}{\alpha+\beta}\text{, with: }\tilde{s}_i=q_i\frac{\alpha^2\left(\lambda_3+\tilde{\phi}_2\right)\tilde{\Phi}}{n}\sum_{i=1}^n\left(\varepsilon_i\bar{\mu}_i^s\right)$$

Same calculations apply, allowing computing the social loss function:

$$\begin{aligned}EL^U\left(r^c\right)=&\frac{1}{2\alpha^2}\left[1-a_1\tilde{\Phi}^2\left[a+2\left(a_2\phi_1-a_1\alpha^2\left(\lambda-\tilde{\lambda}\right)\right)\right]\right]\sigma_\mu^2+\frac{\lambda_2}{2a_1}\sigma_{\bar{\mu}}^2\\&-\left[a\left(\tilde{\lambda}_3+\tilde{\phi}_2\right)-2\lambda_3\left(\tilde{a}+a_2\tilde{\phi}_1\right)\right]\frac{\alpha^2\left(\tilde{\lambda}_3+\tilde{\phi}_2\right)\tilde{\Phi}^2}{2a_1}E\left[\left(\frac{1}{n}\sum_{i=1}^n\left(\varepsilon_i\bar{\mu}_i^s\right)\right)^2\right]\end{aligned} \quad \text{(A4)}$$

Using the fact that: $\tilde{\lambda}_3+\tilde{\phi}_2=\lambda_3+\phi_2$, one can easily find expression (15') in the main text, on the two above mentioned assumptions,

Welfare differentials $\left(\Delta EL=EL^U\left(r^c\right)-EL^U\left(r^u\right)\right)$

One can find equation (11) of the main text by computing (A1)-(A2):

$$\Delta EL=\left(\frac{\lambda_1a_2^2}{2\alpha^2\lambda_1a_1a}\right)\sigma_\mu^2+\frac{\alpha^2\left(\lambda_3\right)^2}{2a_1a}E\left[\left(\frac{1}{n}\sum_{i=1}^n\left(\varepsilon_i\bar{\mu}_i^s\right)\right)^2\right]>0 \quad \text{(A5)}$$

From (A3)-(A2), we obtain the social-loss differential with aversion to divergences:

$$\Delta EL = \left(\frac{a_1 {a_2}^2 {\phi_1}^2 \Phi^2}{2\alpha^2 a}\right)\sigma_\mu^2 + \frac{1}{2a_1}\left(\frac{\alpha^2 {\lambda_3}^2}{a} - [(a+2a_2\phi_1)\lambda_3 - a\phi_2]\alpha^2(\lambda_3+\phi_2)\Phi^2\right)E\left[\left(\frac{1}{n}\sum_{i=1}^{n}(\varepsilon_i \bar{\mu}_i^s)\right)^2\right]$$

(A6)

from which we extract equation (16) in the main text. Finally, from (A4)-(A2) we find the social-loss differential with aversion to divergences and central bank own preferences:

$$\Delta EL = \left(\frac{a_1}{2\alpha^2 a}\left[a_2\phi_1 - \alpha^2 a_1\left(\lambda-\tilde{\lambda}\right)\right]^2 \tilde{\Phi}^2\right)\sigma_\mu^2 + \frac{\alpha^2}{2a}\left[(\alpha+\beta)\left[\lambda_1\phi_2 + \alpha^2\lambda_3\left(\lambda-\tilde{\lambda}\right)\right] + \alpha a_2\phi_3\right]^2 \tilde{\Phi}^2 E\left[\left(\frac{1}{n}\sum_{i=1}^{n}(\varepsilon_i\bar{\mu}_i^s)\right)^2\right] \quad \text{(A7)}$$

which corresponds to equation (16') in the main text.

Relations (11), (16) and (16') are computed owning to the assumption: $E\left[\left(\frac{1}{n}\sum_{i=1}^{n}(\varepsilon_i\bar{\mu}_i^s)\right)^2\right] = \Sigma^2\sigma_{\bar{\mu}}^2$. Relations (A5), (A6) and (A7) are more general and do not depend on assumptions i) and ii) above, thus generalizing our main text findings.

Appendix 2: Introducing demand shocks

Let us suppose now that there are only demand shocks $\left(\mu^s = \bar{\mu}_i^s = 0\right)$. Interest rate responses in different regimes are still general relations (8), (9), (14) and (14'). The expressions of the social loss functions are now[26]:

- *under the centralized regime:*

$$EL^U\left(r^c\right) = Y_d\sigma_\mu^2 + \frac{\lambda_1}{2a_1}\sigma_{\bar{\mu}}^2, \text{ where: } Y_d = \frac{\lambda_1}{2\alpha^2}\left(1 - \frac{a_2 - a_1}{a_1}\right) \quad \text{(B1)}$$

- *under the optimal regime:*

$$EL^U\left(r^u\right) = X_d\sigma_\mu^2 + \frac{\lambda_1}{2a_1}\sigma_{\bar{\mu}}^2 + \bar{X}_d E\left[\left(\frac{1}{n}\sum_{i=1}^{n}(\varepsilon_i\bar{\mu}_i^d)\right)^2\right] \quad \text{(B2)}$$

where: $X_d = \frac{\lambda_1}{2\alpha^2}\left(1 - \frac{a_1}{a}\lambda_1\right)$ and: $\bar{X}_d = -\frac{\alpha^2(\lambda_1)^2}{2a_1 a}$.

[26] Of course variances of shocks now design variances of demand shocks: $\sigma_\mu^2 = \sigma_{\mu^d}^2$ and $\sigma_{\bar{\mu}}^2 = \frac{1}{n}\sum_{i=1}^{n}\sigma_{\bar{\mu}_i^d}^2$.

- *under the centralized regime with aversion towards divergences:*

$$EL^U(r^c) = Z_d \sigma_\mu^2 + \frac{\lambda_1}{2a_1}\sigma_{\bar{\mu}}^2 + \bar{Z}_d E\left[\left(\frac{1}{n}\sum_{i=1}^{n}\left(\varepsilon_i \bar{\mu}_i^d\right)\right)^2\right] \tag{B3}$$

where: $Z_d = \frac{\lambda_1}{2\alpha^2}\left(1 - a_1\lambda_1\left(a + 2a_2\phi_1\right)\Phi^2\right)$ and: $\bar{Z}_d = -\frac{\alpha^2\left(\lambda_1+\phi_1\right)\lambda_1\Phi^2}{2a_1}\left[a + \phi_1\left(a_2 - a_1\right)\right]$

Welfare differentials are obtained by computing respectively (B1)-(B2) and (B3)-(B2). We obtain first the social loss differential between the centralized regime and the optimal one (B1-B2):

$$\Delta EL = EL^U\left(r^c\right) - EL^U\left(r^u\right) = \left(Y_d - X_d\right)\sigma_\mu^2 - \bar{X}_d E\left[\left(\frac{1}{n}\sum_{i=1}^{n}\left(\varepsilon_i \bar{\mu}_i^d\right)\right)^2\right] \tag{B5}$$

Since $\Sigma^2 \leq 1$, thus $a_1 > a_2$, and: $Y_d - X_d = \frac{\lambda_1}{2\alpha^2}\frac{2a_1^{\,2} - a_2^{\,2}}{a_1\left(a_1 + a_2\right)} > 0$. Furthermore: $\bar{X}_d < 0$, thus: $\Delta EL > 0$, and: $EL^U\left(r^c\right) > EL^U\left(r^u\right)$. As for supply shocks, welfare is higher with a 'cooperative' than with a centralized monetary policy.

The social loss differential between the centralized regime with aversion towards divergences and the optimal one (B3-B2) is:

$$\Delta EL = EL^U\left(r^c\right) - EL^U\left(r^u\right) = \left(Z_d - X_d\right)\sigma_\mu^2 + \left(\bar{Z}_d - \bar{X}_d\right)E\left[\left(\frac{1}{n}\sum_{i=1}^{n}\left(\varepsilon_i \bar{\mu}_i^d\right)\right)^2\right] \tag{B6}$$

where: $Y_d - X_d = \frac{\left(a_2\phi_1\Phi\lambda_1\right)^2 a_1}{2\alpha^2 a} \geq 0$ and: $\bar{Z}_d - \bar{X}_d = \frac{\left(\alpha\lambda_1 a_1\phi_1\Phi\right)^2}{2a_1 a} \geq 0$.

Notice that, in the case of demand shocks only, there is infinity of optimal contracts. Effectively, centralized regime with aversion towards divergences becomes efficient if $\phi_1 = 0$, namely if: $\theta_\pi = 1 + \alpha^2\left(\lambda - \theta_y\right)$. This condition ensures stabilizing both symmetric and asymmetric demand shocks in the optimal way. Effectively, with only demand shocks, there is no trade-off between stabilizing inflation and stabilizing output: if $\mu_i^s = 0$ in equation (1), equilibrium output will linearly depend on inflation. Thus, the problem of stabilization becomes very simple (and very specific) and can be solved by all contracts that verify $\phi_1 = 0$.

Since $\phi_1 = 0$ condition is already a necessary, *but not sufficient*, condition for supply shocks to be properly stabilized, supply shocks can be studied separately, as in our main text, without loss of generality.

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