Households access to financial markets and the growth rate of a risky economy

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Abstract

Most developing economies have bank-dominated financial systems and the development of a financial market to finance growth is questionable. In an overlapping generation framework based on the models developed by Fecht et al. (2005) and Marini (2005), we build four different types of financial systems and compare their respective contributions to the growth rate of production in a context of a risky technology. Results show that letting households access the secondary market should not be a priority in a risky economy as intermediation yields higher growth rates of capital and production.
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1 Introduction

Financial systems affect economic growth.[1] It is a crucial question in developed economies and is even more important for less developed countries. Developing economies are indeed seeking ways of financing their economic growth as they strongly depend on external fundings but face a reduction of international aid flows today. The Washington Consensus of the early 1990s defends policies promoting economic growth based on the role of the markets.[2] In fact, liberal approach argues that markets are more effective than banking indebtedness, however, history in developed countries has proved that these two logics of financing (banks and markets) are more complementary than completely opposed.[3] In less developed countries, financial systems are generally bank-dominated and even though banks are excessively liquid, they offer very little credit to the private sector and prefer to invest in Treasury Bonds which finance budget deficits. [4] Reforms of the financial system are therefore a matter of concern and question the opportunity (in terms of economic growth) for countries to develop alternative sources of financing for firms and to develop financial markets.

Empirical models have shown that even if financial development is good for economic growth, the bank or market-based structure is equivalent as what matters is quality of financial services and legal rights (Levine (2002), Allen and Gale (2001)). As a matter of fact, Levine (2004) provides an exhaustive survey on the empirical and theoretical conclusions drawn and yet the debate does not seem closed. We provide an original answer to that traditional question using existing models of financial intermediation.

Theoretical models of financial intermediation are generally based on the seminal paper by Diamond and Dybvig (1983). This paper accounts for intermediaries performing intertemporal risk sharing whereas Allen and Gale (1997, 2003) distinguish two types of risks : intertemporal risk sharing, when risks are shared across time, and cross sectional or aggregate risk sharing when agents exchange risks at a point in time. Nevertheless few models embed this framework in a growth model to account both for the financial structure and its impact on economic growth. Fecht et al.

[1]See Levine (1997) for a survey on this litterature

[2]The Consensus recommends economic reforms such as fiscal discipline, competitive exchange rates, trade liberalisation, foreign investments, privatisations and deregulation.

[3]For instance, in Anglo-Saxon countries, markets dominate while in Germany, banks play a more significant part and in Japan, indebtedness was used to finance growth. See also Schumpeter (1934), Goldsmith (1969), Gurley and Shaw (1955).

[4]For instance, in Madagascar commercial banks hold 84% of financial assets while the remaining 16% are shared between non bank financial institutions which hold 7% of the assets, pension funds holding 4%, microfinance institutions holding 1% and other (institutions) holding 4%. Nevertheless they hold an excess of 24% of the reserve ratio in 2006 and yet their contributions to finance the economy is very low (credit to the private sector representing 15% of total credits in 2006 against a mean of 60% in developed economies). Firms lack fundings (and are undercapitalized) because most investments go in Treasury Bonds (that yield more return and are more reliable) which finance budget deficits.

(2005) compare the performances of markets and intermediaries on investments and growth but omit the analysis of cross sectional risk sharing and do not consider the fact that technology can be risky. Marini (2005) compares the performances of five types of financial systems on growth and welfare but concentrates only on the function of markets used to share aggregate risks without considering the possibility for households to access the financial markets.

In this paper, we reconcile both analyses and study the design of the financial system and how it affects the growth rate of a risky economy.

We use the previous models in an overlappping generation framework and we account for, not only intertemporal but also aggregate risk sharing. We also distinguish our results according to the degree of risk affecting productive technology therefore we can consider the case of a risky economy subject to several shocks. In fact we build four different types of financial systems, from the most basic (where only banks exist) to the more complex (where both banks, markets and capitalists supply funds). Two of them are bank-dominated while the last two are mixed and provide households an access to the financial market as they can become sophisticated.[5] For each category of financial system we discern the case where the bank uses the market to hedge against aggregate risk and the case where the bank does not diversify. We then examine the contributions of each financial system to the growth rate of capital and to the growth rate of production through simulation techniques.

We show that in incomplete markets with low risks, a mixed financial system letting households access the secondary market is better for the growth rate of capital than financial intermediation alone. Nonetheless when risks are high, financial intermediation proves to be better at sharing risks and investing in profitable projects. We also highlight the benefits of cross sectional risk sharing that allows investment in better but riskier projects. Therefore in order to increase the growth rate of production in a risky economy, letting households access the financial market should not be a priority. Reforms should obviously aim at finding more financing resources but these funds should be intermediated.

The rest of the paper is organised as follows. In section 2 we present our general framework, in section 3, the results of our model and in section 4, a numerical example which allows us to compare the different financial systems and draw conclusions in section 5.

[5] Note that in our financial systems we always consider limited market participation for households, that means that banks have access to the primary market and households to the secondary market. Nevertheless, we intend to study the case where households have a direct access to the primary market in future work.

2 An OLG model with financial intermediation

We consider an overlapping generation framework as in Fecht et al. (2005). We nonetheless introduce the possibility to change the characteristics of the financial system of the economy.
Our economy is populated by a mass of 1 risk-averse households each of whom living two periods $t = 1, 2$. There is an initial old generation and we suppose an exogeneous interest rate equal to zero between the periods.
Each agent is subject to an idiosyncratic preference shock and is either an impatient (early) or a patient (late) consumer. We call her respectively type 1 or type 2 agent. Preferences are identical and given by

$$U(c_t) = \begin{cases} u(c_1) \text{ with probability } q, \\ u(c_2) \text{ with probability } 1 - q. \end{cases}$$

The fraction of type 1 agents is then q and there is $(1 - q)$ type 2 agents.
$U(.)$ is a von Neumann Morgenstein (CRRA) utility function and c_t is consumption at date t.

$$u(c_t) = \frac{c_t^{1-\alpha}}{1-\alpha}.$$

$\alpha > 1$ is the constant relative risk aversion and the utility function is twice continuously differentiable, increasing, strictly concave and satisfies the Inada conditions.[6]
Each period is divided into two subperiods corresponding to the beginning and the end of the period.

The economy consists of four types of agents : households, entrepreneurs, banks and a representative risk neutral capitalist. Their presence and activities in the economy may change depending on the composition of the financial system we study. We assume the absence of bank runs and the absence of intergenerationnal risk sharing. There is only one consumption good in each period.

At the beginning of period one, old agents own a stock of K_t units of capital.[7] This is rented out to firms that employ young households endowed with one unit of labour when young and nothing when old. Production takes place.

Entrepreneurs use the following production function $Y_t = \overline{K}_t^{1-\theta} L_t^{1-\theta} K_t^{\theta}$ where $\overline{K}_t$ is the average capital-labour ratio in the economy at time t, hence a positive capital externality on productivity.[8]

[6] $u'(0) = \infty$ et $u'(\infty) = 0$.
[7] with a stock of K_1 units of capital for the initial old generation.
[8] As in Fecht et al. (2005) and Ennis and Keister (2003), this is a common assumption in AK- growth models and is one way of generating endogeneous growth. The exponent on this term is explained by Antinofi and al (2001).

Perfect competition in the factor markets is assumed and labour is supplied inelastically. The equilibrium real wage and the real capital rental rate are then respectively given by $w_t = (1-\theta)K_t$ and $v_t = \theta$.
Households deposit their entire income in the bank with the following deposit contract (r_{1t}, r_{2t}) without knowing if they are patient or impatient.[9]

The perfectly competitive bank uses part of the deposits to buy the existing undepreciated capital $(1-\delta)K_t$ from old agents at the price $P_t^- = \frac{1}{R}$ which is the equilibrium price in units of the consumption good in the primary market of capital.[10] Note that for old households willing to rent their capital to firms before selling it to banks, it must be $v_t \geq \delta P_t^-$.
The rest of the deposits placed in the bank is divided between storage and investment in new capital. We suppose that only banks purchase existing capital or invest in new capital or storage.

One unit of consumption placed in storage at time t yields one unit of consumption when it is liquidated (at any moment or period t) while one unit of consumption placed in investment in period t yields $R > 1$ units of capital in $t+1$. The productive technology is the only way to produce new capital[11].

Nevertheless, we take into account the fact that technology can be risky so the return on investment is contingent on the state of nature in $t+1$.[12] The bank can then face insolvency if the return on its investment in the long asset is unsuccessful. This represents an aggregate shock regarding the return on investment in the long asset in $t+1$. Thus, each unit of consumption invested at t yields a random return of $R(s)$ units of capital, where $s \in S$ is a state of nature at the beginning of $t+1$. There are two possible states of nature denoted H and L, with $R(H) > R(L)$. The probabilities are $P(H)$ and $P(L)$ where $P(s)$ is a common prior probability density for all agents over the states of nature. Nonetheless, the expected return on investment is greater than the return on storage $E(\widetilde{R}) = R(H)P(H) + R(L)P(L) \geq 1$.

At the end of period one, young households realize if they are impatient and prefer

[9] (r_{1t}, r_{2t}) represent the repayments the bank provides to its impatient and patient depositors.

[10] It is the price of capital in the beginning of the first period and it can be shown that it is the price at which banks are indifferent between buying existing capital or investing in new capital

[11] Investment in the long term asset is then illiquid and yields a lower return $x < 1$ (units of consumption) than storage when liquidated early

[12] For example, when production is risky and the economy faces different types of shocks such as climatic shocks (cyclones, droughts) which have serious consequences on production and growth if the economy strongly depends on agricultural income or political shocks or external shocks like changes in international prices of raw materials (for developing economies) or changes in trade agreements and so on.

to consume now to obtain r_{1t}, while they are still young, or patient and then delay their consumption r_{2t} for the period 2 when they will be old. We assume that this can be public or private information.
For each unit of capital owned by an old agent at the beginning of the period, she obtains therefore $\left[v_t + (1-\delta)P_t^-\right] K_t$. She consumes this and exits the economy.
Not considering the possibility of two states of nature, the return of capital in each period is : $X = R\left[v_t + (1-\delta)P_t^-\right] = R\left[\theta + (1-\delta)R_t^{-1}\right] = R\theta + 1 - \delta$ with $X > 1$ and $v_t \geq \delta P_t^-$.[13]

In period two, young agents from period 1 become old and impatient ones consume. The old ones disappear and a new set of young generation is born.

3 Different financial systems and first results

Here, we compare four different types of financial systems to analyse their contributions to economic growth. We want to see the differences between financial systems dominated by banks (where only these latter have access to investment) and financial systems allowing participation of households in the financial market. We categorize our financial systems on this criteria.

- S1 and S2 are systems in which only banks have access to long term investment. We call them "bank only" systems.
- S3 and S4 are systems in which banks have access to the primary market of capital but these systems also allow households participation in the secondary market of capital. We call them mixed systems.

Nevertheless, we also take into account the fact that the bank can share aggregate risk (and undertake cross sectional risk sharing) or not.

Table 1: Characteristics of the financial systems.

	Bank only	Mixed
No cross sectional risk sharing	**S1**	**S3**
Cross sectional risk sharing	**S2**	**S4**

We therefore start by analyzing the most basic financial system (S1) and proceed to the more complex one in order to examine the contributions of households access to the financial market.
This implies that our framework differs according to the system considered. We first present the characteristics of our four systems before identifying the results they yield.

[13]One unit of capital rented yields v_t. The undepreciated capital is then sold (according to $v_t \geq \delta P_t^-$) at the price P_t^- and all the proceeds from renting and selling the capital are invested to produce new capital in the next period.

3.1 S1 - The basic financial system

In this financial system, intermediation prevails. There is no market and the bank is the only agent who has access to investment. We refer here to the mutual bank without financial markets (FS1) described by Marini (2005) but incorporate it to the OLG framework.

The bank collects households savings and invests these in storage or in the long term productive asset (investment for new capital in $t+1$).[14] It issues a demand deposit contract and can be interpreted as a coalition of risk-averse agents. We assume a perfectly competitive bank and because entry in the banking sector is free, the bank always maximizes the utility of its representative depositor.

As two states of nature exist in 2, the bank has two budget constraints depending on the state of nature. It issues an incentive compatibility constraint to make sure that depositors reveal their preferences and stick to them (so patient households do not claim to be impatient).

Optimisation yields the following consumptions (see appendix for details),

$$r_{1t} = \frac{w_t \epsilon}{q\epsilon + \frac{1-q}{X(H)}},$$

$$r_{2tH} = \frac{w_t}{q\epsilon + \frac{1-q}{X(H)}},$$

$$r_{2tL} = \frac{X(L)}{X(H)} r_{2tH},$$

with $\epsilon = \left[P(H) + P(L)\left(\frac{X(L)}{X(H)}\right)^{1-\alpha}\right]^{\frac{1}{1-\alpha}}$.

If the state of nature in $t = 2$ is L then depositors will only get a fraction of $\frac{X(L)}{X(H)}$ of repayments they get when s is H.

It is possible to derive from these **the amount invested in the productive technology**. The bank is the only agent who invests in the long term technology. As in Fecht et al. (2005), the repayments to impatient depositors represent what is invested in storage as these households do not have access to investment. However, the amount that the bank needs to make the repayments to patient depositors in $t = 2$ represents what is invested in the productive technology. This depends on the state of nature in $t = 2$ as the technology is risky,

$$K^1_{t+1} = (1-q)P(H)\frac{R(H)}{X(H)} r_{2tH} + (1-q)P(L)\frac{R(L)}{X(L)} r_{2tL}.$$

[14]In fact, households prefer to deposit their income in the bank which will make the investment.

When replacing r_{2tH} and r_{2tL} and w_t we find,

$$K^1_{t+1} = \frac{(1-q)\left[P(H)R(H) + P(L)R(L)\right]}{q\epsilon X(H) + 1 - q}(1-\theta)K^1_t$$

with $\epsilon = \left[P(H) + P(L)\left(\frac{X(L)}{X(H)}\right)^{1-\alpha}\right]^{\frac{1}{1-\alpha}}$.

3.2 S2 - The bank and the capitalist

In S1, the bank only ensure intertemporal risk-sharing (see Diamond and Dybvig (1983)). We know that the type 1 or 2 of depositors are public information, therefore to hedge aggregate risk, the bank must use the market. A set of Arrow-Debreu securities is then exchanged by the bank and the capitalist. We refer here to the mutual bank with financial markets (FS2) described by Marini (2005) but incorporate it to the OLG framework.

The risk-neutral capitalist exchanges these securities with the bank to allow aggregate risk sharing as the existence of different states of nature is public information. Therefore the capitalist does not undertake production and we do not calculate general equilibrium.[15] A representative capitalist is endowed with C units of capital and knows that she will consume only in period 2. She is not subject to a preference shock and maximizes her consumption in period 2. Households do not interact with the capitalist.

There are two different types of Arrow-Debreu securities and their prices are defined as,

- $P_2(H)$ if $s = H$, *i.e.* the number of units of the goods needed at 1 to buy the promise that one unit of the good will be delivered in period 2 if $s = H$ (and zero if not),
- and $P_2(L)$ if $s = L$, *i.e.* the number of units of the goods needed at 1 to buy the promise that one unit of the good will be delivered in period 2 if $s = L$ (and zero if not).

The set of Arrow-Debreu securities is defined for all possible states of nature, therefore there is only one intertemporal budget constraint for the bank.

The bank maximizes the expected utility of its representative depositor subject to its intertemporal budget constraint and an incentive compatibility to keep its patient depositors until $t = 2$.

[15]Marini(2005) studies general equilibrium.

The returns of the deposit contract are

$$r_{1t} = \left[P(H)\Psi^{\frac{1-\alpha}{\alpha}} + P(L)\right]^{\frac{1}{1-\alpha}} r_{2tL},$$

$$r_{2tH} = r_{2tL}\Psi^{\frac{1}{\alpha}},$$

$$r_{2tL} = \frac{w_t}{q\left[P(H)\Psi^{\frac{1-\alpha}{\alpha}} + P(L)\right]^{\frac{1}{1-\alpha}} + (1-q)P_2(H)\Psi^{\frac{1}{\alpha}} + (1-q)P_2(L)},$$

with $\Psi = \frac{P_2(L)P(H)}{P_2(H)P(L)}$.

The amount invested in the productive technology is, as in S1, given by the total refinancing the bank needs to repay the patient depositors who decide to withdraw at $t = 2$. The difference with respect to the previous system lies in the Arrow-Debreu securities through diversification and aggregate risk sharing.

$$K^2_{t+1} = (1-q)P(H)\frac{R(H)}{X(H)}r_{2tH} + (1-q)P(L)\frac{R(L)}{X(L)}r_{2tL}.$$

By replacing r_{2tH} and r_{2tL},

$$K^2_{t+1} = \frac{(1-q)\left[\frac{R(L)P(L)}{X(L)} + \frac{R(H)P(H)}{X(H)}\psi^{\frac{1}{\alpha}}\right]}{q\left[P(H)\psi^{\frac{1-\alpha}{\alpha}} + P(L)\right]^{\frac{1}{1-\alpha}} + (1-q)\left[P_2(H)\psi^{\frac{1}{\alpha}} + P_2(L)\right]}(1-\theta)K^2_t$$

with $\psi = \frac{P_2(L)P(H)}{P_2(H)P(L)}$.

3.3 S3 - The bank and the market

Households may also have the opportunity to participate in the secondary market of capital and buy the capital from the bank at the equilibrium price $P^+_t = \frac{1}{R}$. To that purpose, households can become sophisticated or remain unsophisticated. Being sophisticated enables them to monitor the entrepreneurs and ensures a return $R > 1$ of their investments, however, this implies an exogeneous utility cost of $(\chi - 1)$.[16] They make this decision at the same time they become aware of the bank deposit contract (r_{1t}, r_{2t}). Unsophisticated households can not monitor entrepreneurs, therefore the latter will always shirk when financed by unsophisticated agents and provide them with the return γR with $R > 1 > \gamma R > 0$. Therefore in our model $\gamma X < 1$. We suppose there exists a fraction of $(1 - i)$ sophisticated depositors called type A and i unsophisticated depositors called type B. This information remains private.

[16]This can also be measured in terms of resources as in Fecht (2005). It can be the cost of acquiring a new degree or the cost of learning for instance.

We refer to the system studied by Fecht et al. (2005) but in our framework we consider two states of nature in period $t+1$.

The types of the depositors , sophisticated (A) or unsophisticated (B), are private information and therefore the bank can not offer a separate deposit contract. There is free entry in the banking sector, the bank can not retain any resources and is the only way for unsophisticated depositors to access to the long term technology as it uses their funds to make all its repayments. The bank therefore maximizes the expected utility of its unsophisticated depositors. The bank must also ensure that type A households want to withdraw early (to participate in the secondary market of capital) but not type B. Consumptions must then verify the incentive compatibility constraints.

We drop the indexes for i_t and ϵ_t because they are the same across the periods. The returns of the deposit contract are,

$$r_{1t} = \frac{w_t \psi}{(1+qi-i)\psi + \frac{(1-q)i\left(\frac{1+qi-i}{qi}\right)^{\frac{1}{\alpha}}}{X(H)}}$$

$$r_{2tH} = \left(\frac{1+qi-i}{qi}\right)^{\frac{1}{\alpha}} \frac{w_t}{(1+qi-i)\psi + \frac{(1-q)i\left(\frac{1+qi-i}{qi}\right)^{\frac{1}{\alpha}}}{X(H)}},$$

$$r_{2tL} = \frac{X(L)}{X(H)} r_{2tH},$$

with $\psi = (X(H))^{\frac{-1}{\alpha}} \left[P(H) + P(L)\left(\frac{X(L)}{X(H)}\right)^{1-\alpha}\right]^{\frac{-1}{\alpha}}$.

These values must verify the incentive compatibility constraints.

- ***For type A depositors***, the incentive compatibility constraint ensures that their investment return in the secondary market in $t=1$ is greater than the return they could obtain by keeping their deposits in the bank. The latter can monitor firms therefore they insure an amount R for each unit invested, thus the return on capital is X. As two states of nature exist in $t=2$, X depends on the probability of these states as well as the return r_{2t} that the bank offer to depositors in $t=2$.

  Replacing the consumptions by their optimal values yields the following condition :

$$i \geq \frac{\left[P(H) + \frac{r_{2tL}}{r_{2tH}} P(L)\right]^{\alpha}}{qX^{\alpha}\psi^{\alpha} + (1-q)\left[P(H) + \frac{r_{2tL}}{r_{2tH}} P(L)\right]^{\alpha}},$$

  with $\psi = (X(H))^{\frac{-1}{\alpha}} \left[P(H) + P(L)\left(\frac{X(L)}{X(H)}\right)^{1-\alpha}\right]^{\frac{-1}{\alpha}}$.

- ***For type B depositors***, we apply the same reasoning so that the Incentive Compatibility (IC) constraint ensures that the return of their investments in the secondary market, where they can not monitor entrepreneurs, is lower that what the bank offer them in $t = 2$. Therefore,

$$i \leq \frac{\left[P(H) + \frac{r_{2tL}}{r_{2tH}} P(L)\right]^{\alpha}}{qX^{\alpha}\psi^{\alpha} + (1-q)\left[P(H) + \frac{r_{2tL}}{r_{2tH}} P(L)\right]^{\alpha}},$$

with $\psi = (X(H))^{\frac{-1}{\alpha}} \left[P(H) + P(L)\left(\frac{X(L)}{X(H)}\right)^{1-\alpha}\right]^{\frac{-1}{\alpha}}$. We have by assumption $\gamma X < 1$ and $X > 1$ then $\gamma X < X$, therefore IC_B is always satisfied.

The amount invested in the productive technology includes the amount necessary to make the repayments to type B patient depositors in $t = 2$ and the amount the bank needs for type A depositors in the secondary market of capital as the latter invest in the productive technology.

$$K^3_{t+1} = (1-q)(1-i)\frac{r_{1t}}{P^+_t} + (1-q)i\frac{r_{2tH}}{X(H)}P(H)R(H) + (1-q)i\frac{r_{2tL}}{X(L)}P(L)R(L).$$

By replacing r_{1t}, r_{2tH} and r_{2tL},

$$K^3_{t+1} = R(1-q)(1-\theta)K^3_t \left[\frac{i\left(\frac{1+qi-i}{qi}\right)^{\frac{1}{\alpha}} + (1-i)\psi X(H)}{(1+qi-i)\psi X(H) + (1-q)i\left(\frac{1+qi-i}{qi}\right)^{\frac{1}{\alpha}}}\right],$$

with $\psi = (X(H))^{\frac{-1}{\alpha}} \left[P(H) + P(L)\left(\frac{X(L)}{X(H)}\right)^{1-\alpha}\right]^{\frac{-1}{\alpha}}$.

3.4 S4 - The bank, the capitalist and the market

In this financial system, the bank uses the market to share aggregate risk with the capitalist and all sophisticated households have access to the secondary market of capital.[17] As in the previous financial systems, the capitalist does not undertake production and does not interact with households. As in S3, there are two types of households A and B and this remains private information.

Therefore, the bank maximizes the expected utility of its unsophisticated depositors, it has a unique intertemporal budget constraint and uses two incentive compatibility constraints to reveal preferences.

[17] It remains, however, a limited market participation.

Solving the banking deposit contract yields the following consumptions,

$$r_{2tH} = \frac{wt}{\left(\frac{P_2(H)}{P(H)}\right)^{\frac{1}{\alpha}} \left[(1+qi-i)\left(\frac{qi}{1+qi-i}\right)^{\frac{1}{\alpha}} + i(1-q)P_2(L)\left(\frac{P(L)}{P_2(L)}\right)^{\frac{1}{\alpha}}\right] + i(1-q)P_2(H)},$$

$$r_{1t} = \left(\frac{qi}{1+qi-i}\right)^{\frac{1}{\alpha}} \left(\frac{P_2(H)}{P(H)}\right)^{\frac{1}{\alpha}} r_{2tH},$$

$$r_{2tL} = \left(\frac{P(L)P_2(H)}{P_2(L)P(H)}\right)^{\frac{1}{\alpha}} r_{2tH}.$$

These must satisfy the incentive compatibility constraints,

- For type A depositors, $r_{1t}\left[P(H)X(H) + P(L)X(L)\right] \geq P(H)r_{2tH} + P(L)r_{2tL}$. And by replacing r_{1t}, r_{2tH} and r_{2tL},

$$i \geq \frac{\left[P(H) + \frac{r_{2tL}}{r_{2tH}}P(L)\right]^{\alpha}}{qX^{\alpha}\left(\frac{P_2(H)}{P(H)}\right) + (1-q)\left[P(H) + \frac{r_{2tL}}{r_{2tH}}P(L)\right]^{\alpha}}.$$

- For type B depositors, $r_{1t}\gamma\left[P(H)X(H) + P(L)X(L)\right] \leq P(H)r_{2tH} + P(L)r_{2tL}$. Thus

$$i \leq \frac{\left[P(H) + \frac{r_{2tL}}{r_{2tH}}P(L)\right]^{\alpha}}{q\gamma^{\alpha}X^{\alpha}\left(\frac{P_2(H)}{P(H)}\right) + (1-q)\left[P(H) + \frac{r_{2tL}}{r_{2tH}}P(L)\right]^{\alpha}}.$$

  Since $\gamma X < 1$, IC_B is always satisfied.

The amount invested in the productive technology follows the same previous reasoning.

$$K^4_{t+1} = (1-q)i\frac{r_{2tH}R(H)}{X(H)}P(H) + (1-q)i\frac{r_{2tL}R(L)}{X(L)}P(L) + (1-q)(1-i)\frac{r_{1t}}{P^+_t},$$

$$K^4_{t+1} = \frac{(1-q)i\left[\frac{R(H)P(H)}{X(H)} + \frac{R(L)P(L)}{X(L)}\left(\frac{P_2(H)P(L)}{P_2(L)P(H)}\right)^{\frac{1}{\alpha}}\right] + R(1-q)(1-i)\left(\frac{qi}{1+qi-i}\right)^{\frac{1}{\alpha}}\left(\frac{P_2(H)}{P(H)}\right)^{\frac{1}{\alpha}}}{(1+qi-i)\left(\frac{qi}{1+qi-i}\right)^{\frac{1}{\alpha}}\left(\frac{P_{2H}}{P(H)}\right)^{\frac{1}{\alpha}} + (1-q)i\left[P_2(H) + P2(L)\left(\frac{P(L)P_2(H)}{P(H)P_2(L)}\right)^{\frac{1}{\alpha}}\right]}(1-\theta)K^4_t,$$

with $R = \left[P(H)R(H) + P(L)R(L)\right]$.

4 Simulation and comparison

We compare the implications of the four previous financial systems with respect to the growth rate they can achieve. We have defined the law of motion for capital in each financial system and we are now able to constrast the growth rates of capital through simulation techniques. Considering the form of our production function, we have $\frac{dY_t}{Y_t} = \theta \frac{dK_t}{K_t}$, which means that we can directly compare the growth rate of capital to look at the growth rate of production.

4.1 Transformation

We first standardize the parameters used in the expressions for capital. For that purpose we make different assumptions on the returns on investment when there exists different possible states of nature. We base our assumptions on the riskiness of the technology.
Let the returns contingent on states of nature H or L be $R(H) = \phi R$, $R(L) = \frac{1}{\phi} R$, with $R(H) > R(L) > 0$ then $\phi \geq 1$.
ϕ can be interpreted as a premium on investment. With the above expressions, we assume that when ϕ increases, the gap between the different returns (contingent on the state of nature) relative to the mean return increases. In this case, $P(H) = \frac{1}{\phi+1}$ and $P(L) = 1 - P(H) = \frac{\phi}{\phi+1}$ and therefore $\frac{dP(L)}{d\phi} = \frac{1}{(\phi+1)^2} > 0$ and $\frac{dP(H)}{d\phi} = \frac{-1}{(\phi+1)^2} < 0$.
We can then conclude that when ϕ increases, the probability that the low state of nature will occur increases too and therefore the technology is more risky but more profitable. Thus variations of ϕ allow us to consider the rise of risks in the economy.

4.1.1 For S1

Replacing the expressions of $R(H)$, $R(L)$, $P(H)$, $P(L)$, $X(H)$ and $X(L)$ yields,

$$\rho_1 = \frac{K_{t+1}^1}{K_t^1} = \frac{R(1-q)(1-\theta)}{q\left[\frac{1}{\phi+1} + \frac{\phi}{\phi+1}\left(\frac{R\theta\frac{1}{\phi}+1-\delta}{R\theta\phi+1-\delta}\right)^{1-\alpha}\right]^{\frac{1}{1-\alpha}}[R\theta\phi + 1 - \delta] + 1 - q}.$$

4.1.2 For S2

We make the same transformations for the above parameters.
Nonetheless, we still have to define the prices $P_2(H)$ and $P_2(L)$ of the Arrow-Debreu securities. We therefore make additional assumptions.

Complete markets and solvent bank We suppose that whatever happens in period 2, the bank is solvent, then $r_{2tH} = r_{2tL}$ and $\left[\frac{P(H)P_2(L)}{P(L)P_2(H)}\right]^{\frac{1}{\alpha}} = 1$.
We also assume the absence of arbitrage opportunities in the financial markets then $P_2(H)X(H) + P_2(L)X(L) = 1$ and that the market is complete, meaning that Arrow-Debreu securities exist for each state of nature.[18]
Under these assumptions, we can then define the prices $P_2(H) = \frac{1}{R(\phi+1)} = \frac{1}{R}P(H)$ and $P_2(L) = \frac{\phi}{R(\phi+1)} = \frac{1}{R}P(L)$.

The growth rate of capital in S2 when markets are complete can be expressed as,

$$\rho_2 = \frac{K_{t+1}^2}{K_t^2} = \frac{(1-q)(1-\theta)\frac{R\phi}{\phi+1}\left(\frac{1}{R\theta+(1-\delta)\phi} + \frac{1}{R\theta\phi+1-\delta}\right)}{q+(1-q)\frac{1}{R}}.$$

Incomplete markets In the previous case, we considered the case of complete markets with banking solvency. It means that it is more expensive to be insured against the low state of nature as $P_2(L)$ is superior to $P_2(H)$ and $r_{2tH} \neq r_{2tL}$. Now we suppose that markets are incomplete. There exists only one type of securities that the bank can exchange with the capitalist and there is no arbitrage oppportunities in the market therefore the price of the security exchanged in the incomplete markets is $P_2(L) = P_2(H) = P_2 = \frac{1}{R}$.
In this case, the expression for the growth rate of capital in S2 is,

$$\rho_2 = \frac{K_{t+1}^2}{K_t^2} = \frac{(1-q)(1-\theta)\left(\frac{R\phi}{\phi+1}\right)\left[\frac{1}{R\theta+(1-\delta)\phi} + \frac{1}{R\phi\theta+1-\delta}\left(\frac{1}{\phi}\right)^{\frac{1}{\alpha}}\right]}{q\left(\frac{1}{\phi+1}\right)\left[\phi+\left(\frac{1}{\phi}\right)^{\frac{1-\alpha}{\alpha}}\right]^{\frac{1}{1-\alpha}} + (1-q)\left(\frac{1}{R}\right)\left[1+\left(\frac{1}{\phi}\right)^{\frac{1}{\alpha}}\right]}.$$

4.1.3 For S3

The growth rate of capital in S3 can be expressed as,

[18] Absence of arbitrage opportunities means that the cost of investing in the financial markets is equal to the cost of investing in the long term assets in the beginning of the period 1. In fact, investing 1 in the long term asset in period 1 yields $X(H)$ if $s = H$ and $X(L)$ if $s = L$ whereas using the financial market costs $P_2(H)X(H) + P_2(L)X(L)$. Therefore, to make sure that no arbitrage opportunities appear, these two costs must be equal.

$$\rho_3 = \frac{K^3_{t+1}}{K^3_t} = R(1-q)(1-\theta)$$

$$\left[\frac{i\left(\frac{1+qi-i}{qi}\right)^{\frac{1}{\alpha}} + (1-i)(R\theta\phi+1-\delta)^{\frac{\alpha-1}{\alpha}}\left[\frac{1}{\phi+1}+\frac{\phi}{\phi+1}\left(\frac{R\theta\frac{1}{\phi}+1-\delta}{R\theta\phi+1-\delta}\right)^{1-\alpha}\right]^{\frac{-1}{\alpha}}}{(1+qi-i)(R\theta\phi+1-\delta)^{\frac{\alpha-1}{\alpha}}\left[\frac{1}{\phi+1}+\frac{\phi}{\phi+1}\left(\frac{R\theta\frac{1}{\phi}+1-\delta}{R\theta\phi+1-\delta}\right)^{1-\alpha}\right]^{\frac{-1}{\alpha}} + (1-q)i\left(\frac{1+qi-i}{qi}\right)^{\frac{1}{\alpha}}} \right],$$

with $i \geq \frac{(R\theta\phi+1-\delta)\left(\frac{1}{\phi+1}+\frac{\phi}{\phi+1}\left(\frac{R\theta\frac{1}{\phi}+1-\delta}{R\theta\phi+1-\delta}\right)^{\alpha-1}\right)}{q[R\theta+1-\delta]^{\alpha}\left(\frac{1}{\phi+1}+\frac{\phi}{\phi+1}\left(\frac{R\theta\frac{1}{\phi}+1-\delta}{R\theta\phi+1-\delta}\right)\right)^{\alpha}+(1-q)(R\theta\phi+1-\delta)\left(\frac{1}{\phi+1}+\frac{\phi}{\phi+1}\left(\frac{R\theta\frac{1}{\phi}+1-\delta}{R\theta\phi+1-\delta}\right)^{\alpha-1}\right)}$.

4.1.4 For S4

As in S2 we distinguish two cases : the case of complete markets and the case of incomplete markets. We obtain the expression for the growth rate of capital in S4 and the condition on i to make sure that the incentive compatibility constraint is binding.

Complete markets and solvent bank

$$\rho_4 = \frac{K^4_{t+1}}{K^4_t} = \frac{R(1-q)(1-\theta)\left[\frac{i}{\phi+1}\left(\frac{\phi}{R\theta\phi+1-\delta}+\frac{1}{R\theta\frac{1}{\phi}+1-\delta}\right)+(1-i)\left(\frac{qi}{1+qi-i}\right)^{\frac{1}{\alpha}}\left(\frac{1}{R}\right)^{\frac{1}{\alpha}}\right]}{(1+qi-i)\left(\frac{qi}{1+qi-i}\right)^{\frac{1}{\alpha}}\left(\frac{1}{R}\right)^{\frac{1}{\alpha}}+(1-q)i\left(\frac{1}{R}\right)},$$

with $i \geq \frac{1}{q(R\theta+1-\delta)^{\alpha}\left(\frac{1}{R}\right)+1-q}$.

Incomplete markets

$$\rho_4 = \frac{K^4_{t+1}}{K^4_t} = \frac{R(1-q)(1-\theta)\left[\frac{i}{\phi+1}\left(\frac{\phi}{R\theta\phi+1-\delta}+\frac{\phi^{\frac{1}{\alpha}}}{R\theta\frac{1}{\phi}+1-\delta}\right)+(1-i)\left(\frac{qi}{1+qi-i}\right)^{\frac{1}{\alpha}}\left(\frac{\phi+1}{R}\right)^{\frac{1}{\alpha}}\right]}{(1+qi-i)\left(\frac{qi}{1+qi-i}\right)^{\frac{1}{\alpha}}\left(\frac{\phi+1}{R}\right)^{\frac{1}{\alpha}}+(1-q)i\left(\frac{1}{R}\right)\left(1+\phi^{\frac{1}{\alpha}}\right)},$$

with $i \geq \frac{\left(\frac{1+\phi^{\frac{1}{\alpha}+1}}{\phi+1}\right)^{\alpha}}{q(R\theta+1-\delta)^{\alpha}\left(\frac{1}{R}\right)+(1-q)\left(\frac{1+\phi^{\frac{1}{\alpha}+1}}{\phi+1}\right)^{\alpha}}$.

4.2 Simulation

To compare the growth rate of capital in each financial system, we use simulation techniques and observe the variations of the rate across different values of ϕ. This numerical example helps us identify the contributions of each financial system to the growth rate.

We take q, $1-q$, α and R as constants and vary them to account for changes in the fraction of impatient households, changes in the intensity of risk aversion and changes in the return on investment. [19] Parameters of the production function are standard and are the same as those used by Fecht et al. (2005). $\theta = 0.33$ and the rate of depreciation is $\delta = 0.96$ corresponding to an average depreciation of capital of 10% per year in 30 years. To make sure that old households are willing to rent their capital to firms before selling it to banks, we must have $\theta R > \delta$ then $R \geq 3$. Because $i < 1$ in our IC constraints, we must have $R \geq 9$. We choose $R = 10$ and $R = 30$ (which nonetheless corresponds to very high growth rates of capital).

Table 2 summarizes our results. They are robust to changes in the fraction of patient depositors, to changes in the degree of risk aversion and to changes in the degree of economic development (with a larger R for developing economies). Details of our numerical results are provided in the appendix and **Figures 1 and 2** represent variations of the growth rate of capital.

Table 2: Results

	ϕ **small**	ϕ **high**
Complete markets	S4>S3>S1>S2	S1>S3>S4>S2
Incomplete markets	S2>S3>S4>S1	S2>S1>S3>S4

We draw the following conclusions with respect to the results of **Table 2**.

Result 1- *Complex financial systems do not always generate higher growth rates of capital than systems in which only banks have access to investment.*

We show that the degree of market completeness and the degree of risk bearing upon production, influence the growth rate of capital and production. We can re-synthesize our results in **Table 3**.

Result 2 - *In complete markets, where all securities needed to be insured against the different states of nature exist and when the bank is solvent, diversification reduces the growth rate of capital.*

This means that when markets are complete, sharing aggregate risks with the capitalist is redundant (and reduces the opportunities to invest in projects that seem more risky but can yield higher returns). Intermediaries are not necessary. This result is

[19] Increases in R allow us to account for the case of developing countries where marginal productivity of capital is supposed to be higher than in developed economies.

Table 3: Results (2)

	low risk mixed>bank	**high risk bank>mixed**
Complete markets - S2 worst	S3>S1>S2	S1>S3>S2
Incomplete markets - S2 best	S2>S3>S1	S2>S1>S3

in line with the traditional theory of financial intermediation explaining that intermediaries are only needed to compensate for the imperfections of markets otherwise they become superfluous. However in incomplete markets aggregate risk sharing is more important than letting households access to the secondary market. Acemoglu and Zilibotti (1997) explain that the desire to avoid highly risky investment reduces capital accumulation (as agents seek insurance by investing in safe but less productive assets) and the ability to diversify idiosyncatic risks decrease uncertainty in the growth process.

Indeed, **Figure 2** shows that in S2 (intermediation with aggregate risk sharing) with complete markets, a higher degree of risk reduces the growth rate of capital whereas with incomplete markets, a higher probability for the low state of nature shows the importance of risk sharing as growth rate of capital increases.

Result 3 - *We also see that when risks are high in the economy, "bank only" systems can increase the growth rate of capital and the growth rate of production more than mixed systems.*

In fact, **Figure 1** shows that the growth rate of capital in S1 (complete intermediation without cross sectional risk sharing) increases as risks increase. Households can contribute to investment (because they have access to the secondary market) but they can not monitor entrepreneurs appropriately without enduring an utility cost and thus the latter always shirk if households are not sophisticated. Intermediaries are the only agent capable of controlling their investment without costs. This result is in line with those of the literature which underlines the functions performed by intermediaries in the financial system and their significance on economic growth. In fact the risk sharing effect and other advantages conferred to bank intermediation outbalance the leverage effect of riskier but more profitable technology. Intermediaries, such as banks, are able to collect and produce information, they can monitor firms at lesser costs than other agents, collect and stimulate savings (by offering attractive instruments) and thus can better allocate savings by granting credits to the best firms, they facilitate exchanges (by providing payment, settlement, clearing and netting services), improve liquidity and share risks. Therefore developing a secondary market intended to households is not a priority for the growth rate in a risky economy. It means that intermediaries can achieve more in terms of capital and production growth than markets.

However when risks are low, a mix of investment from the bank and from households is better for the growth rate of production. Granting access to the secondary market to sophisticated households allow the identification of new profitable opportunities and financing based on profitability if banks are powerful familial conglomerates for instance. This allows funds go to the most productive uses (which are generally the most risky).
For example, if we take the case of an economy traditionnally subject to international shocks, as domestic production grows, it could increase its domestic demand for money and be less dependent on external resources. That will undoubtedly diminish the impact of external shocks on the domestic economy and the degree of risks on production. In the end, as it develops a stronger domestic production base, moving towards a more mixed financial system could help finance growth. That means that in the process of development, economies tend to move towards financial markets.

S4 position is ambiguous as it is a mixed system that implement cross-sectional risk sharing and nonetheless grant households access to the financial market. We see from **Table 1** that an arbitrage arises between the two effects. The dominating effect is not clear as it depends on the degree of risks bearing upon production and completeness of the market. A closer look into the characteristics of our financial systems shows that if S3 generates a higher growth rate than S4, it means that households access to the secondary market of capital yields more growth, whereas when S4 is superior to S3 it suggests that risk sharing is more important. Likewise, comparing S2 and S4 or S2 and S3 inform on what seems to matter more for growth rates: risk sharing or households access to the secondary market of capital. Taking all these into account allows us to come to the following conclusions that are summarized in **Table 4**.

Table 4: Results (3)

	low risk **mixed>bank**	**high risk** **bank>mixed**
Complete markets - S2 worst	S4>S3>S1>S2	S1>S3>S4>S2
S4 position explains what matters more	**households access**	**households access**
Incomplete markets - S2 best	S2>S3>S4>S1	S2>S1>S3>S4
S4 position explain what matters more	**diversification**	**diversification**

Result 4 - *A financial system implementing risk sharing achieves a higher growth rate of capital than a system trying to increase financing of firms by letting households access the secondary market without adequate risk sharing.*

Our previous conclusions stating the importance of cross sectional risk sharing, are confirmed. We see that in incomplete markets, risk sharing matters more than households access to the secondary market of capital whereas when markets are complete, households access to the market increases the growth rate of capital by about 4 points when $R = 10$ and risks are high (see **Table 5 and Table 6**) as securities insuring against different states of nature already exist.

Intermediaries absorb risks and manage risks in incomplete markets as they permit a diversified portfolio of investments.
Even though intermediation is more appropriate in a risky economy (subject to risky technology), this arbitrage, between the risk sharing effect and the growth enhancing effect of letting households access the market, is satisfied whatever the degree of risk bearing upon productive technology.

We also note that an increase in the fraction of impatient depositors *i.e.* in the number of early consumers that prefer to consume while they are still young, diminishes the growth rate of production.[20] Increases in the degree of risk aversion also reduces the growth rate while increases of R, meaning considering the case of a developing economy, increases the growth rate of capital. All these results are consistent with the litterature on economic growth.

5 Conclusions

Cross sectional risk sharing helps overcome difficulties that arise because of risky investments. Therefore an economy should not consider households access to the financial markets as a priority to finance growth if production is risky. It should aim at developing at first other sources of financing through financial intermediaries. Nonetheless, as the economy grows and becomes less dependent on external resources and less vulnerable to shocks, letting households access the financial market could help finance growth.
In future work, we intend to enrich our results by considering the case where households can have a direct access to both the primary and the secondary market. Finally, we would like to directly compare the growth rates of capital and production and the degree of risk sharing provided in each financial system.

[20]Impatient depositors can be sophisticated or not. To make sure that sophisticated depositors reveal their preferences (and invest), we had the Incentive Compatibility constraint imposing a large number of unsophisticated households (see IC for S3 and S4).

References

Acemoglu, D. and Zilibotti, F. (1997). Was prometheous unbound by chance? risk, diversification and growth. *Journal of Political Economy*, 105:709–751.

Allen, F. and Gale, D. (2001). Comparative financial systems: A survey.

Allen, F. and Gale, D. (2003). Financial intermediaries and markets.

Diamond, D. and Dybvig, P. (1983). Bank runs, deposit insurance and liquidity. *Journal of Political Economy*, 91:401–419.

Ennis, H. M. and Keister, T. (2003). Economic growth, liquidity, and bank runs. *Journal of Economic Theory*, 109:220–245.

F.Allen and D.Gale (1997). Financial markets, intermediaries, and intertemporal smoothing. *Journal of Political Economy*, 105(3):523–46.

Fecht, F., Huang, K., and Martin, A. (2005). Financial intermediairies, markets and growth.

Goldsmith, R. (1969). Financial structure and development. *New Haven CT: Yale University Press*.

J.Gurley and G.Shaw (1955). Financial aspects of economic development. *American Economic Review*, 45(4):515Ű538.

Levine, R. (2002). Bank-based or market-based financial systems: Which is better? *Journal of Financial Intermediation*, 11(4):398–428.

Levine, R. (2004). Finance and growth : theory and evidence.

Marini, F. (2005). Banks, financial markets, and social welfare. *Journal of banking and finance*, 29:2557–2575.

R.Levine (1997). Financial development and economic growth : views and agenda. *Journal of Economic Literature*, 35(2):688–726.

Schumpeter, J. (1934). The theory of economic development. *Harvard University Press*.

Appendix

The bank problem

In S1

The bank solves the following problem :

$$\begin{cases} \max\limits_{r_{1t}, r_{2tH}, r_{2tL}} & q\frac{(r_{1t})^{1-\alpha}}{1-\alpha} + (1-q)P(H)\frac{(r_{2tH})^{1-\alpha}}{1-\alpha} + (1-q)P(L)\frac{(r_{2tL})^{1-\alpha}}{1-\alpha} \\ \text{s.t.} & qr_{1t} + (1-q)\frac{r_{2Ht}}{X(H)} = w_t, \\ & qr_{1t} + (1-q)\frac{r_{2tL}}{X(L)} = w_t, \\ & \frac{(r_{1t})^{1-\alpha}}{1-\alpha} \leq P(H)\frac{(r_{2tH})^{1-\alpha}}{1-\alpha} + P(L)\frac{(r_{2tL})^{1-\alpha}}{1-\alpha}. \end{cases} \tag{1}$$

The budget constraints always bind in the optimal deposit contract. They yield : $r_{2tH} = \frac{X(H)}{X(L)} r_{2tL}$ and $r_{2tL} = \frac{X(L)}{X(H)} r_{2tH}$. Replacing r_{2tL} in the program yields

$$\begin{cases} \max\limits_{r_{1t}, r_{2tH}} & q\frac{(r_{1t})^{1-\alpha}}{1-\alpha} + (1-q)P(H)\frac{(r_{2tH})^{1-\alpha}}{1-\alpha} + (1-q)P(L)\frac{\left(\frac{X(L)}{X(H)}r_{2tH}\right)^{1-\alpha}}{1-\alpha} \\ \text{s.t.} & qr_{1t} + (1-q)\frac{r_{2Ht}}{X(H)} = w_t, \\ & \frac{(r_{1t})^{1-\alpha}}{1-\alpha} \leq P(H)\frac{(r_{2tH})^{1-\alpha}}{1-\alpha} + P(L)\frac{\left(\frac{X(L)}{X(H)}r_{2tH}\right)^{1-\alpha}}{1-\alpha}. \end{cases}$$

All constraints are linear. We use the Kuhn-Tucker conditions.
The Lagrangean is :

$$\begin{aligned} L = q\frac{(r_{1t})^{1-\alpha}}{1-\alpha} + (1-q)P(H)\frac{(r_{2tH})^{1-\alpha}}{1-\alpha} + (1-q)P(L)\frac{\left(\frac{X(L)}{X(H)}r_{2tH}\right)^{1-\alpha}}{1-\alpha} \\ +\lambda\left[qr_{1t} + (1-q)\frac{r_{2tH}}{X(H)} - w_t\right] \\ +\mu\left[\frac{(r_{1t})^{1-\alpha}}{1-\alpha} - P(H)\frac{(r_{2tH})^{1-\alpha}}{1-\alpha} - P(L)\frac{\left(\frac{X(L)}{X(H)}r_{2tH}\right)^{1-\alpha}}{1-\alpha}\right]. \end{aligned}$$

First order conditions are :

- $\frac{\partial L}{\partial r_{1t}} = 0$,
- $\frac{\partial L}{\partial r_{2tH}} = 0$,
- $\frac{\partial L}{\partial \lambda} = 0$,
- $\frac{\partial L}{\partial \mu} \leq 0$,

- $\mu \left[\frac{(r_{1t})^{1-\alpha}}{1-\alpha} - P(H)\frac{(r_{2tH})^{1-\alpha}}{1-\alpha} - P(L)\frac{\left(\frac{X(L)}{X(H)}r_{2tH}\right)^{1-\alpha}}{1-\alpha} \right] = 0$,
- $\mu \geq 0$.

If the incentive compatibility constraint is binding It means that all constraints are binding at the optimum, therefore we have $\frac{\partial L}{\partial \mu} = 0$ and $\mu \neq 0$. Optimisation yields

$$r_{1t} = \frac{w_t \epsilon}{q\epsilon + \frac{1-q}{X(H)}},$$

$$r_{2tH} = \frac{w_t}{q\epsilon + \frac{1-q}{X(H)}},$$

$$r_{2tL} = \frac{X(L)}{X(H)} r_{2tH},$$

with $\epsilon = \left[P(H) + P(L) \left(\frac{X(L)}{X(H)}\right)^{1-\alpha} \right]^{\frac{1}{1-\alpha}}$.

If the incentive compatibility constraint is not binding The budget constraint is the only binding constraint. Therefore $\mu = 0$ and $\frac{\delta L}{\delta \mu} > 0$. Optimisation yields

$$r_{1t} = \frac{w_t \psi}{q\psi + \frac{1-q}{X(H)}}$$

$$r_{2tH} = \frac{w_t}{q\psi + \frac{1-q}{X(H)}},$$

$$r_{2tL} = \frac{X(L)}{X(H)} r_{2tH},$$

with $\psi = (X(H))^{\frac{-1}{\alpha}} \left[P(H) + P(L) \left(\frac{X(L)}{X(H)}\right)^{1-\alpha} \right]^{\frac{-1}{\alpha}}$.

NB : In the calculus for the law of motion for capital we made the assumption that the incentive compatibility constraint is binding. It means that the return on withdrawing in t=1 and the return on withdrawing in t=2 are equal.

The consumptions r_{2tH} vary depending on the fact that the incentive compatibility constraint is binding or not. However, the fraction of r_{1t} and r_{2tL} in r_{2tH} remains the same.

In S2

Intertemporal budget constraint in S2 Let y_B be the fraction of deposits invested in storage, z_B, the fraction of deposits invested in capital (old and new), and $\eta_B(s)$: the number of Arrow-Debreu securities bought by the bank.

First, the bank uses the deposits w_t in storage, in capital and in Arrow-Debreu securities (to share risks). It faces the following budget constraint $y_B w_t + z_B w_t + P_2(H)\eta_B(H) + P_2(L)\eta_B(L) = w_t$
When impatient depositors withdraw their deposits, it must be $qr_{1t} = y_B w_t$.
In $t = 2$, depending on the state of nature, $(1-q)r_{2tH} = X(H)z_B w_t + \eta_B(H)$ and $(1-q)r_{2tL} = X(L)z_B w_t + \eta_B(L)$.
The bank has an unique intertemporal budget constraint as Arrow-Debreu securities exist for each state of nature. Then we obtain $qr_{1t} + z_B w_t\left[1 - P_2(H)X(H) - P_2(L)X(L)\right] + (1-q)\left[P_2(H)r_{2tH} + P_2(L)r_{2tL}\right] = w_t$. The absence of arbitrage opportunities yields $P_2(H)X(H) + P_2(L)X(L) = 1$. Therefore, in equilibrium

$$qr_{1t} + (1-q)\left[P_2(H)r_{2tH} + P_2(L)r_{2tL}\right] = w_t.$$

The bank problem in S2

$$\left\{\begin{array}{ll} \max\limits_{r_{1t}, r_{2tH}, r_{2tL}} & q\frac{(r_{1t})^{1-\alpha}}{1-\alpha} + (1-q)\frac{P(H)(r_{2tH})^{1-\alpha}}{1-\alpha} + (1-q)\frac{P(L)(r_{2tL})^{1-\alpha}}{1-\alpha} \\ \text{s.t.} & qr_{1t} + (1-q)r_{2tH}P_2(H) + (1-q)r_{2tL}P_2(L) = w_t, \\ & \frac{P(H)(r_{2tH})^{1-\alpha}}{1-\alpha} + \frac{P(L)(r_{2tL})^{1-\alpha}}{1-\alpha} \geq \frac{(r_{1t})^{1-\alpha}}{1-\alpha}. \end{array}\right. \tag{2}$$

All constraints are linear. We use the Kuhn-Tucker conditions.
The Lagrangean is :

$$\begin{aligned} L = {} & q\frac{(r_{1t})^{1-\alpha}}{1-\alpha} + (1-q)P(H)\frac{(r_{2tH})^{1-\alpha}}{1-\alpha} + (1-q)P(L)\frac{r_{2tL}^{1-\alpha}}{1-\alpha} \\ & +\lambda\left[qr_{1t} + (1-q)P_2(H)r_{2tH} + (1-q)P_2(L)r_{2tL} - w_t\right] \\ & +\mu\left[P(H)\frac{(r_{2tH})^{1-\alpha}}{1-\alpha} + P(L)\frac{(r_{2tL})^{1-\alpha}}{1-\alpha} - \frac{(r_{1t})^{1-\alpha}}{1-\alpha}\right]. \end{aligned}$$

First order conditions are :

- $\frac{\partial L}{\partial r_{1t}} = 0$,
- $\frac{\partial L}{\partial r_{2tH}} = 0$,
- $\frac{\partial L}{\partial r_{2tL}} = 0$
- $\frac{\partial L}{\partial \lambda} = 0$,
- $\frac{\partial L}{\partial \mu} \geq 0$,
- $\mu\left[P(H)\frac{(r_{2tH})^{1-\alpha}}{1-\alpha} + P(L)\frac{(r_{2tL})^{1-\alpha}}{1-\alpha} - \frac{(r_{1t})^{1-\alpha}}{1-\alpha}\right] = 0$,
- $\mu \geq 0$.

If the incentive compatibility constraint is binding It means that all constraints are binding at the optimum, therefore we have $\frac{\partial L}{\partial \mu} = 0$ and $\mu \neq 0$. Optimisation yields

$$r_{1t} = \left[P(H)\Psi^{\frac{1-\alpha}{\alpha}} + P(L)\right]^{\frac{1}{1-\alpha}} r_{2tL},$$

$$r_{2tH} = r_{2tL}\Psi^{\frac{1}{\alpha}},$$

$$r_{2tL} = \frac{w_t}{q\left[P(H)\Psi^{\frac{1-\alpha}{\alpha}} + P(L)\right]^{\frac{1}{1-\alpha}} + (1-q)P_2(H)\Psi^{\frac{1}{\alpha}} + (1-q)P_2(L)},$$

with $\Psi = \frac{P_2(L)P(H)}{P_2(H)P(L)}$.

If the incentive compatibility constraint is not binding The budget constraint is the only binding constraint. Therefore $\mu = 0$ and $\frac{\delta L}{\delta \mu} > 0$. Optimisation yields

$$r_{1t} = \frac{w_t}{q + (1-q)\left(P(H)^{\frac{1}{\alpha}}P_2(H)^{\frac{\alpha-1}{\alpha}} + P(L)^{\frac{1}{\alpha}}P_2(L)^{\frac{\alpha-1}{\alpha}}\right)},$$

$$r_{2tH} = \left(\frac{P(H)}{P_2(H)}\right)^{\frac{1}{\alpha}} r_{1t},$$

$$r_{2tL} = \left(\frac{P(L)}{P_2(L)}\right)^{\frac{1}{\alpha}} r_{1t}.$$

In S3

The budget constraint In $t = 1$, the bank repays all impatient depositors who withdraw and patient sophisticated depositors. Then $[q(1-i) + qi + (1-q)(1-i)]r_{1t} + (1-q)i\frac{r_{2t}}{X} = w_t$. Therefore, we get $[qi + (1-i)]\, r_{1t} + (1-q)i\frac{r_{2t}}{X} = w_t$.

The bank problem in S3 The bank solves the following program,

$$\begin{cases} \max\limits_{r_{1t},r_{2tH},r_{2tL}} & qi\frac{(r_{1t})^{1-\alpha}}{1-\alpha} + (1-q)P(H)i\frac{(r_{2tH})^{1-\alpha}}{1-\alpha} + (1-q)P(L)i\frac{(r_{2tL})^{1-\alpha}}{1-\alpha} \\ \text{s.t.} & [1+qi-i]r_{1t} + (1-q)i\frac{r_{2Ht}}{X(H)} = w_t, \\ & [1+qi-i]r_{1t} + (1-q)i\frac{r_{2tL}}{X(L)} = w_t, \\ & r_{1t}\left[P(H)X(H) + P(L)X(L)\right] \geq P(H)r_{2tH} + P(L)r_{2tL}, \\ & r_{1t}\gamma\left[P(H)X(H) + P(L)X(L)\right] \leq P(H)r_{2tH} + P(L)r_{2tL}. \end{cases} \tag{3}$$

The budget constraint always binds in the optimal deposit contract. We note that $r_{2tL} = \frac{X(L)}{X(H)}r_{2tH}$. Therefore the bank-maximisation problem subject to the budget constraint solves,

$$\begin{cases} \max_{r_{1t}, r_{2tH}, r_{2tL}} & qi\frac{(r_{1t})^{1-\alpha}}{1-\alpha} + (1-q)P(H)i\frac{(r_{2tH})^{1-\alpha}}{1-\alpha} + (1-q)P(L)i\frac{\left(\frac{X(L)}{X(H)}r_{2tH}\right)^{1-\alpha}}{1-\alpha} \\ \text{s.t.} & [1+qi-i]r_{1t} + (1-q)i\frac{r_{2Ht}}{X(H)} = w_t. \end{cases}$$

The Lagrangean is :

$$L = qi\frac{(r_{1t})^{1-\alpha}}{1-\alpha} + (1-q)i\frac{(r_{2tH})^{1-\alpha}}{1-\alpha}\left[P(H) + P(L)\left(\frac{X(L)}{X(H)}\right)^{1-\alpha}\right]$$
$$+\lambda\left[(1+qi-i)r_{1t} + (1-q)i\frac{r_{2tH}}{X(H)} - w_t\right].$$

First order conditions are :

- $\frac{\partial L}{\partial r_{1t}} = 0,$
- $\frac{\partial L}{\partial r_{2tH}} = 0,$
- $\frac{\partial L}{\partial \lambda} = 0.$

The consumptions are therefore,

$$r_{1t} = \frac{w_t\psi}{(1+qi-i)\psi + \frac{(1-q)i\left(\frac{1+qi-i}{qi}\right)^{\frac{1}{\alpha}}}{X(H)}}$$

$$r_{2tH} = \left(\frac{1+qi-i}{qi}\right)^{\frac{1}{\alpha}} \frac{w_t}{(1+qi-i)\psi + \frac{(1-q)i\left(\frac{1+qi-i}{qi}\right)^{\frac{1}{\alpha}}}{X(H)}},$$

$$r_{2tL} = \frac{X(L)}{X(H)}r_{2tH},$$

with $\psi = (X(H))^{\frac{-1}{\alpha}}\left[P(H) + P(L)\left(\frac{X(L)}{X(H)}\right)^{1-\alpha}\right]^{\frac{-1}{\alpha}}$.

These values must verify the incentive compatibility constraints.

- ***For type A depositors***, replacing the consumptions by their optimal values yields the following condition :

$$i \geq \frac{1}{q\left(\frac{X\psi}{\left[P(H)+P(L)\left(\frac{X(L)}{X(H)}\right)\right]}\right)^{\alpha} + 1 - q} \Leftrightarrow i \geq \frac{\left[P(H) + \frac{r_{2tL}}{r_{2tH}}P(L)\right]^{\alpha}}{qX^{\alpha}\psi^{\alpha} + (1-q)\left[P(H) + \frac{r_{2tL}}{r_{2tH}}P(L)\right]^{\alpha}},$$

with $\psi = (X(H))^{\frac{-1}{\alpha}}\left[P(H) + P(L)\left(\frac{X(L)}{X(H)}\right)^{1-\alpha}\right]^{\frac{-1}{\alpha}}$.

- ***For type B depositors***, it must be

$$i \leq \frac{\left[P(H) + \frac{r_{2tL}}{r_{2tH}} P(L)\right]^{\alpha}}{qX^{\alpha}\psi^{\alpha} + (1-q)\left[P(H) + \frac{r_{2tL}}{r_{2tH}} P(L)\right]^{\alpha}},$$

with $\psi = (X(H))^{\frac{-1}{\alpha}} \left[P(H) + P(L)\left(\frac{X(L)}{X(H)}\right)^{1-\alpha}\right]^{\frac{-1}{\alpha}}$. We have by assumption $\gamma X < 1$ and $X > 1$ then $\gamma X < X$, therefore IC_B is always satisfied.

In S4

We follow the same reasoning as in S2 and S4 for the intertemporal budget constraint of the bank.
We therefore solve the following program:

$$\begin{cases} \max\limits_{r_{1t}, r_{2tH}, r_{2tL}} & qi\frac{(r_{1t})^{1-\alpha}}{1-\alpha} + (1-q)i\frac{(r_{2tH})^{1-\alpha}}{1-\alpha}P(H) + (1-q)i\frac{(r_{2tL})^{1-\alpha}}{1-\alpha}P(L) \\ \text{s.t.} & [1+i(q-1)]\, r_{1t} + (1-q)ir_{2tH}P_2(H) + (1-q)ir_{2tL}P_2(L) = wt, \\ & r_{1t}\left[P(H)X(H) + P(L)X(L)\right] \geq P(H)r_{2tH} + P(L)r_{2tL}, \\ & r_{1t}\gamma\left[P(H)X(H) + P(L)X(L)\right] \leq P(H)r_{2tH} + P(L)r_{2tL}. \end{cases} \tag{4}$$

As in S3, we first solve the bank problem subject to the budget constraint. We use the Lagrangean.
The Lagrangean is :

$$L = qi\frac{(r_{1t})^{1-\alpha}}{1-\alpha} + (1-q)iP(H)\frac{(r_{2tH})^{1-\alpha}}{1-\alpha} + (1-q)iP(L)\frac{r_{2tL}^{1-\alpha}}{1-\alpha}$$
$$+\lambda\left[(1+i(q-1))r_{1t} + (1-q)iP_2(H)r_{2tH} + (1-q)iP_2(L)r_{2tL} - w_t\right].$$

First order conditions are :

- $\frac{\partial L}{\partial r_{1t}} = 0,$
- $\frac{\partial L}{\partial r_{2tH}} = 0,$
- $\frac{\partial L}{\partial r_{2tL}} = 0$
- $\frac{\partial L}{\partial \lambda} = 0.$

We therefore find the following consumptions:

$$r_{2tH} = \frac{wt}{\left(\frac{P_2(H)}{P(H)}\right)^{\frac{1}{\alpha}}\left[(1+qi-i)\left(\frac{qi}{1+qi-i}\right)^{\frac{1}{\alpha}} + i(1-q)P_2(L)\left(\frac{P(L)}{P_2(L)}\right)^{\frac{1}{\alpha}}\right] + i(1-q)P_2(H)},$$

$$r_{1t} = \left(\frac{qi}{1+qi-i}\right)^{\frac{1}{\alpha}} \left(\frac{P_2(H)}{P(H)}\right)^{\frac{1}{\alpha}} r_{2tH},$$

$$r_{2tL} = \left(\frac{P(L)P_2(H)}{P_2(L)P(H)}\right)^{\frac{1}{\alpha}} r_{2tH}.$$

They must satisfy the incentive compatibility constraints. It is easy to derive the conditions on i by replacing r_{1t}, r_{2tH} and r_{2tL} in the defined conditions.

Simulation results

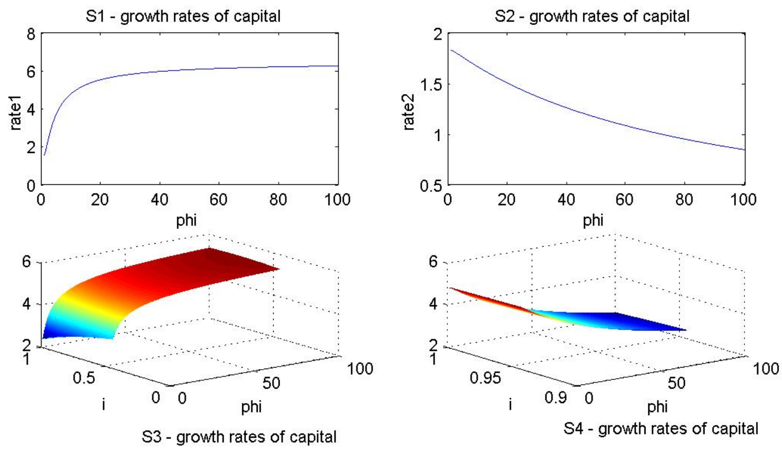

Figure 1: *Complete markets and bank solvency*

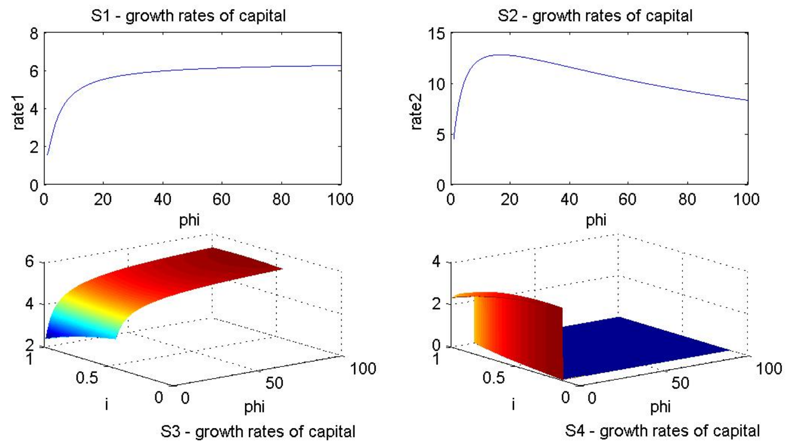

Figure 2: *Incomplete markets*

Table 5: Maximum growth rates of the capital stock across ϕ when $\alpha = 2$ and $R = 10$

$\alpha = 2$, $R = 10$							
	Complete markets			**Incomplete markets**			
S1		$\phi = 1$	$\phi = 2$		$\phi = 100$		
q=0.2		3.6512	4.2888		6.5787		
q=0.5		1.5438	2.0623		6.2399		
q=0.7		0.7619	1.0725		5.7165		
S2	$\phi = 1$	$\phi = 2$	$\phi = 100$	$\phi = 1$	$\phi = 2$	$\phi = 17$	$\phi = 100$
q=0.2	5.7314	5.6977	2.6538	7.6419	9.2106	12.9471	8.2777
q=0.5	1.8236	1.8129	0.8444	4.4578	6.6973	12.7458	8.2726
q=0.7	0.8244	0.8195	0.3817	2.5608	4.5096	12.403	8.2636
S3			**low ϕ**		**high ϕ**		
q=0.2			5.5027		6.375		
q=0.5			3.5107		5.3088		
q=0.7			2.1238		4.1014		
S4	**low ϕ**		**high ϕ**		**low ϕ**		**high ϕ**
q=0.2	11.2028		11.2028		6.7175		7.9622
q=0.5	4.8892		4.8892		3.2274		3.6792
q=0.7	2.4658		2.4658		1.7168		1.8828

Table 6: Maximum growth rates of the capital stock across ϕ when α increases

$\alpha = 4$, $R = 10$							
	Complete markets			**Incomplete markets**			
S1			$\phi = 1$	$\phi = 2$	$\phi = 100$		
q=0.2			3.6512	4.2888	6.5787		
q=0.5			1.5438	2.0623	6.2399		
q=0.7			0.7619	1.0725	5.7165		
S2	$\phi = 1$	$\phi = 2$	$\phi = 100$	$\phi = 1$	$\phi = 2$	$\phi = 17$	$\phi = 100$
q=0.2	5.7314	5.6977	2.6538	7.6419	9.2106	12.9471	8.2777
q=0.5	1.8236	1.8129	0.8444	4.4578	6.6973	12.7458	8.2726
q=0.7	0.8244	0.8195	0.3817	2.5608	4.5096	12.403	8.2636
S3			**low** ϕ		**high** ϕ		
q=0.2			5.4246		6.4764		
q=0.5			3.3965		5.8729		
q=0.7			2.0388		5.0432		
S4	**low** ϕ		**high** ϕ	**low** ϕ		**high** ϕ	
q=0.2	11.2028		11.2028	6.7175		7.9622	
q=0.5	4.8892		4.8892	3.2274		3.6792	
q=0.7	2.4658		2.4658	1.7168		1.8828	

Table 7: Maximum growth rates of the capital stock across ϕ when R increases

$\alpha = 2$, $R = 30$							
	Complete markets			**Incomplete markets**			
S1			$\phi = 1$	$\phi = 2$	$\phi = 100$		
q=0.2			3.6512	4.2888	6.5787		
q=0.5			1.5438	2.0623	6.2399		
q=0.7			0.7619	1.0725	5.7165		
S2	$\phi = 1$	$\phi = 2$	$\phi = 100$	$\phi = 1$	$\phi = 2$	$\phi = 17$	$\phi = 100$
q=0.2	5.7314	5.6977	2.6538	7.6419	9.2106	12.9471	8.2777
q=0.5	1.8236	1.8129	0.8444	4.4578	6.6973	12.7458	8.2726
q=0.7	0.8244	0.8195	0.3817	2.5608	4.5096	12.403	8.2636
S3			**low** ϕ		**high** ϕ		
q=0.2			16.3215		18.8497		
q=0.5			10.2505		14.8793		
q=0.7			6.1583		10.7609		
S4	**low** ϕ		**high** ϕ	**low** ϕ		**high** ϕ	
q=0.2	11.2028		11.2028	6.7175		7.9622	
q=0.5	4.8892		4.8892	3.2274		3.6792	
q=0.7	2.4658		2.4658	1.7168		1.8828	