Asset prices and information disclosure under recency-biased learning

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Abstract: Much of the earlier literature on how to avoid bubbles in international financial markets has focused on the role of monetary policy and macro-prudential regulation. Nevertheless, taking into account the financial stability objective in the monetary policy design may induce trade-offs with other monetary policy objectives. On the reverse, this paper focuses on the role of information disclosure in a consumption-based asset pricing model in which fluctuations in asset prices are persistently driven by backward-looking expectations due to learning on the fundamental process from agents who weight more recent observations relative to older ones. When the regulator knows the true fundamental process, perfect information disclosure straightforwardly eliminates non-fundamental fluctuations in asset prices. However, as highlighted by various commentators of the recent financial crisis in 2007-2008, the regulator might also not know the true process and be recency-biased. I investigate the consequences of this assumption on the efficiency of information disclosure and identify under which conditions on the regulator learning process information disclosure could have contributed to significantly reduce the booms and busts episodes in the US S&P 500 price index in the run-up to the recent financial crisis and its aftermath.

JEL classification: G15, G12, D83, D84.

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1 Introduction

During the early 2000s Australian housing and credit bubble, the Reserve Bank of Australia implemented original ‘open mouth operations’ in order to mitigate the extent of the steep increase in asset prices and leverage ratios. This communication policy aimed at warning economic agents against the risk that this increase was not driven by fundamental factors and thus not sustainable, and might end up in a very costly bust. To this aim, officials from the RBA made public statements highlighting such concern (see Bloxham et al. (2010) for more details) from mid 2002 onwards. This ‘public awareness campaign’ (Bloxham et al. (2010)), combined with other tools such as monetary policy and regulation tightening, proved rather successful, as the boom ended up in late 2003.

This example suggests that communication policy on the risk of a costly bust when a bubble is identified on a specific financial market –that is when the price deviates from its fundamental value– could be an alternative measure to other standard tools directed at slowing the pace of the increase in prices. The need for such an alternative matters all the more so than much debate subsists on whether monetary policy should consider an asset prices stability objective when setting its interest rate, even after the global financial crisis. Thus, Mishkin (2011) and Woodford (2012), among others, argue that including a financial stability objective in the monetary policy reaction function may generate trade-offs with standard monetary policy objectives, which provides incentives to look for other tools to manage non-fundamental asset prices bubbles. In particular, Williams (2014) emphasizes the role of subjective expectations in explaining fluctuations in asset prices, suggesting that information disclosure and communication policy aiming at impacting these expectations and bringing them closer to their rational counterparts could be a relevant alternative tool.

Nevertheless, it remains very hard to identify bubbles due to the difficulty to deter-
mine the fundamental value of an asset. Therefore, even the regulator (a financial market regulation authority or a central bank for instance), might not know the true underlying fundamental process of the economy, just as economic agents. It might not be exempt from several biases when forming its expectations or making its optimal decision. Andrew Haldane, a Chief Economist with the Bank of England, highlighted that the design of stress tests in the banking sector in the early 2000s suffered from short memory: they only took into account the distribution of main macroeconomic and financial variables in the most recent decade—the ‘Golden Decade’, characterized by very low variance—whereas it was very distinct from the long-run historical distribution (Haldane (2009)). Academic literature has also put recent emphasis on the biases the regulator faces when making its decision (Cooper and Kovacic (2012)).

Therefore, this paper argues that information disclosure can help mitigate non-fundamental fluctuations in asset prices under specific conditions on the regulator learning process on the true fundamental distribution. It does so in a standard asset pricing model with recency-biased Bayesian learning on the true parameters of the fundamental process.

Indeed, it is now well known that standard asset pricing models under rational expectations are unable to replicate several stylized facts observed in the stock market. On the contrary, recent macro-finance literature shows that relaxing the rational expectations assumption—the assumption that agents know the true laws of motion of the economy—under several forms proves very successful in explaining various long-standing empirical puzzles in asset pricing theory (Adam et al. (2015)). In this framework, agents learn the true parameters of the economy laws of motion by updating their estimates after observing new realizations of the economic outcome. Thus, this literature emphasizes how variations in backward-looking expectations
can generate endogenous fluctuations in financial markets. Several empirical elements tend to prove that expectations in financial markets are indeed backward-looking. For instance, Malmendier and Nagel (2011) show that individuals who have experienced periods of deep stress in financial markets are more pessimistic on the future evolution of the stock market than individuals who have not. Similarly, de Bondt and Thaler (1985) show that stocks of firms which performed badly over the prior years are undervalued whereas stocks of firms which performed well are overvalued.

This paper draws upon literature on the role of learning in explaining asset prices and models a simple endowment economy in the spirit of the Lucas’ tree model (1978), in which agents optimally allocate their post-consumption wealth between a risky asset and a risk-free asset. In each period, dividends on the risky asset are drawn from a log-normal distribution. The rational expectations version of this model is first presented as a benchmark tool and then compared to a setting in which agents no longer know the mean of the dividend growth process and learn it through adaptive Bayesian learning. An additional ingredient of the learning process is the myopic behavior of agents regarding the past: they discount more older realizations of the dividend growth than more recent ones in their inference process. Agents are thus ‘recency-biased’, as suggested by empirical evidence (de Bondt and Thaler (1990)). In the face of this empirical evidence, this assumption has become rather standard in theoretical literature as well (Bansal and Shaliastovich, 2010), (Nakov and Nuno, 2015). In this framework which emphasizes the role of expectations in explaining non-fundamental fluctuations in asset prices, the focus of the paper is on the informative role of a regulator aiming at stabilizing expectations on the underlying fundamental process in order to reduce fluctuations in asset prices around their fundamental value. Just as the economic agents, the regulator is assumed not to know the true law of motion of the economy and its information on the true parameters is given endogenously by its
own possibly recency-biased learning process. The model is calibrated on the US S&P 500 index (Shiller’s dataset) over the period 1960-2014, as a structural break in the US dividend process in 1960 was identified by Pesaran et al. (2007). Whereas several learning models proved successful in replicating the long-run moments and evolution in the US price-dividend ratio but missed the recent period, the model presented here replicates better its qualitative recent evolutions. Indeed, Adam et al. (2015)’s model with learning on prices replicates very well the behavior of the US price-dividend ratio from the end of WWII up to the bust of the dot-com bubble, but displays very significant differences with the data on the recent period 2003-2014. Similarly, thanks to a model of learning on stock dividends and stock prices, Nakov and Nuno (2015) replicate well the US price-dividend ratio from 1920 onwards but miss the last 25 years.

This may suggest—perhaps surprisingly—that during the run-up to the subprime crisis, agents formed more fundamental-based expectations whereas the dot-com bubble rather resulted from self-fulfilling expectations. In addition, the model allows to replicate the autocorrelation in the price-dividend ratio and the predictability of returns by the lagged price-dividend ratio at several horizons in the recent period I focus on.

The learning mechanism in my setting is distinct from those investigated in prior literature. First, it relies on Bayesian learning (slightly modified in order to account for recency bias), which, differently to adaptive learning specifications, is optimal under uncertainty on the true parameter of the underlying fundamental process. Differently to Adam et al. (2015) who introduce Bayesian learning on prices in order to obtain feedback effects, the learning mechanism developed in this paper implies no restriction on the ability of agents to derive the impact of their expectations on prices. This enables me to identify the specific impact of the only bias that
is introduced in my setting: the recency bias. In addition, closed-form solutions for stock prices can be derived in this setting, what enables to make explicit the dependence of the price-dividend ratio on expectations.

The impact of several distinct policies on asset prices volatility was investigated in similar frameworks with expectations-driven booms and busts in prices. Thus, Adam et al. (2014) show that a lump-sum tax on financial transactions may deepen volatility in asset prices even though it mitigates trading volumes. Winkler (2014) argues that relaxing the rational expectations assumption regarding the behaviour of stock prices enables a monetary policy rule which reacts to asset prices to impact those prices.

However, much less attention was devoted to the role of the regulator’s communication policy in a backward-looking expectations setting, and to the endogenous acquisition of information by the regulator through its own –possibly also recency-biased– learning process. Albagli et al. (2013) investigate the role of dispersed information between informed traders and uninformed noise traders on asset prices in a different setting whereas I focus on information disclosure arising from the regulator’s communication policy. In addition, to the best of my knowledge, the implication of the regulator’s recency bias on the efficiency of its information disclosure was never investigated.

This enables me to show that even though communication policy arises as a natural tool to mitigate non-fundamental fluctuations triggered by learning, its efficiency is much limited as soon as the regulator displays even a very small recency bias.

Section 1 presents the standard rational expectations model for benchmark purposes and shows how its results contradict the data. Section 2 derives the subjec-
tive expectations model with Bayesian learning on the location parameter of the dividend growth process and recency bias. Section 3 introduces the regulator’s learning process and information disclosure and its impact on agents’ expectations. Section 4 illustrates the impact of information disclosure depending on the regulator’s recency bias through counterfactual simulations on the US S&P 500 price-dividend ratio in the run-up to the financial crisis. Section 5 concludes.

2 The benchmark rational expectations model

The theoretical setting is a simple endowment economy, drawn upon the Lucas’ tree model (1978). In each period, a representative risk-averse agent with CRRA utility function decides what to consume and what to invest in a risky asset (stock) which pays exogenous perishable dividends and a risk-free asset (bonds) which pays an endogenous interest rate. All quantities are expressed in units of a single consumption good. The representative agent’s maximization program is thus the following:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}$$

s.t.

$$P_tS_t + C_t + B_t = (P_t + D_t)S_{t-1} + R_tB_{t-1},$$

where $\beta$ is the discount factor, $\gamma > 0$ is the relative risk aversion coefficient, $C_t$ is consumption in period $t$, $P_t$ is the stock price, $S_t$ the quantity of stocks, $D_t$ the dividends earned on stocks and $R_t$ the real interest rate on bonds, $B_t$ the quantity of bonds in period $t$ (price normalized to 1). Stocks are in one unit supply (as in the Lucas’ model) and bonds are in zero unit supply.

Therefore, the budgetary constraint reduces to $C_t = D_t$ and the Euler equations write:

$$C_t^{-\gamma} = \beta E_t[C_{t+1}^{-\gamma}Z_{t+1}],$$
with $Z_t = \frac{D_{t+D_t}}{P_{t-1}}$ the gross return on stocks, and

$$C_t^{-\gamma} = \beta R_t + E_t[C_{t+1}^{-\gamma}].$$ \hspace{1cm} (4)

Following the empirical literature (see for instance LeRoy and Parke (1992)), the dividends growth rate follows a log-normal process with parameters $d$ and $\sigma$:

$$\log\left(\frac{D_t}{D_{t-1}}\right) = d + \sigma \varepsilon_t,$$ \hspace{1cm} (5)

with $d > 0$, $\sigma > 0$ and $\varepsilon_t$ white noise.

Consequently, the first Euler equation (with respect to stocks), reduces to:

$$D_t^{-\gamma} = \beta E_t[D_{t+1}(\frac{P_{t+1} + D_{t+1}}{P_t})].$$ \hspace{1cm} (6)

This allows to derive an explicit expression for the price of stocks:

$$P_t = \delta D_t,$$ \hspace{1cm} (7)

where $\delta = \frac{\beta \theta}{1-\beta \theta}$ and $\theta = \exp(d(1-\gamma) + \frac{(1-\gamma)^2 \sigma^2}{2})$ if and only if $\beta \theta < 1$ (see proof in Appendix A).

As stated by Lucas (1978), price is then 'a function of the physical state of the economy'. It is a constant share of current payoffs on stocks. Therefore, fluctuations in prices only reflect dividend shocks. Non-fundamental bubbles cannot arise. This makes any information disclosure unnecessary. We will see below that this no longer holds when one relaxes the rational expectations assumption.

In addition, realized returns on stocks are identically and independently distributed.
Indeed,

\[ R_t = \frac{P_t + D_t}{P_{t-1}} = \frac{1 + \delta}{\delta} \frac{D_t}{D_{t-1}} = \frac{1}{\beta \theta} \exp(d + \sigma \varepsilon_t). \]  

(8)

All these implications of the rational expectations standard asset pricing model are at odds with several features of the data on the US S&P 500, as shown in Appendix B. First, the price-dividend ratio is not constant over time, suggesting that non-fundamental fluctuations in asset prices do arise. Second, realized returns are not identically and independently distributed over time. On the contrary, by allowing agents to learn the location parameter of the logged dividend growth distribution, one can replicate simultaneously these features, which gives room to the role of information disclosure.

3 The subjective expectations model

We now assume that agents no longer know the true location parameter of the logged dividend growth process \( d \) and learn it over time through Bayesian updating.\(^1\)

3.1 Beliefs dynamics

Agents observe the realizations of the random variable \( y_t = \log(\frac{D_t}{D_{t-1}}) \) but not the innovation \( \varepsilon_t \). Agents see \( d \) as a random variable, they take into account the uncertainty on their estimate. At the beginning of each period, they have a prior belief on the distribution of \( d \) that they update following the new realization of the dividend growth that they observe, according to Bayes’ rule. The only de-

\(^1\)The location parameter of the logged dividend growth is the only thing agents have to learn. They know that dividend growth follows a log-normal distribution and they know its precision parameter. Assuming that agents know the true precision parameter allows to identify directly the impact of the evolution in one specific belief on asset prices as agents only learn one parameter. In addition, with standard conjugate priors, the posterior distribution for the variance does not exist as shown in Pesaran et al. (2007).
parture from rational behavior under uncertainty that we impose is that agents are recency-biased due to limited ability to take into account the past. Therefore, when deriving the posterior belief in period $t$, less recent observations are discounted relative to more recent ones with a discount factor $0 \leq \alpha < 1$. Thus, in $t$, observation going back to period $t - k$ is discounted by $\alpha^k$.

Bayes' rule applied in period $t$ writes:

$$P(d \mid I_t) \propto L(y^t \mid d)P(d \mid I_{t-1}),$$

with $P(d \mid I_t)$ the posterior distribution, $P(d \mid I_{t-1})$ the prior distribution and $L(y^t \mid d)$ the likelihood function. $I_t$ is the set of information available at date $t$, which includes the history of discounted realizations of the logged dividend growth $y^t = \{\alpha^{t-1}y_1, \alpha^{t-2}y_2, ..., \alpha y_{t-1}, y_t\}$, $\sigma$ and the fact that the logged dividend growth follows a normal distribution with parameters $d$ and $\sigma$.

Given that the logged dividend growth process follows a normal distribution, a natural prior distribution for $d$ is the normal conjugate prior, which allows to derive the posterior distribution analytically. Under this assumption and with recency-biased Bayesian learning, the prior distribution at the beginning of period $t \geq 1$ is $d \sim N(m_t^p, \sigma_t^p)$ with:

$$m_t^p = \frac{y_t - \alpha \rho + y_{t-1} \alpha^2 \rho + ... + y_1 \alpha^{t-1} \rho + y_0 \alpha^t \rho}{(\alpha + \alpha^2 + ... + \alpha^t) \ast \rho},$$

where $\rho = \frac{1}{\sigma}$, that is $\rho$ is the known precision of the logged dividend process.

$$\sigma_t^p = \frac{1}{(\alpha + \alpha^2 + ... + \alpha^t) \ast \rho}.$$  

Therefore, the posterior distribution at the end of period $t$ is given by $d \sim N(m_t, \sigma_t)$ with:

$$m_t = \frac{y_t + y_{t-1} \alpha \rho + y_{t-2} \alpha^2 \rho + ... + y_0 \alpha^t \rho}{(1 + \alpha + \alpha^2 + ... + \alpha^t) \ast \rho},$$
and

\[ \sigma_t = \frac{1}{(1 + \alpha + \alpha^2 + \ldots + \alpha^t) \cdot \rho}. \]  

(13)

See Fink (1997).

As \( \alpha < 1 \), this reduces to \( \sigma_t = \frac{1}{\rho(1 - \alpha^{t+1})} \). This term represents the uncertainty on the posterior estimate of \( d \) in time \( t \). Unsurprisingly, it decreases in \( \alpha \). When the weight allocated to earlier data is higher, the precision of the signal on the true parameter provided by each past realization of the data is higher.

Under standard Bayesian learning (\( \alpha = 1 \)), uncertainty on the posterior estimate of \( d \) is equal to \( \frac{1}{n \cdot \rho} \) with \( n \) the number of past observations (past observations are not discounted). Therefore, when the sample of past observations increases, uncertainty on the estimate of \( d \) decreases and finally converges to zero. The posterior estimate of \( d \) then converges to its true value. Under recency-biased Bayesian learning (\( \alpha \neq 1 \)), beliefs never converge to rational expectations ones. Indeed,

\[ \lim_{N \to \infty} (1 + \alpha + \alpha^2 + \ldots + \alpha^N) = \frac{1}{1 - \alpha}. \]

Thus, precision on the estimate is never infinite and agents’ estimates continue to evolve following new realizations of logged dividend growth even in the limit.

### 3.2 Stock prices under recency-biased Bayesian learning

Given this set of beliefs, agents make optimal decisions. Their maximization program is still given by equations (1)-(2).\(^2\) Under recency-biased Bayesian learning,\(^2\) Nevertheless, as shown by Pesaran et al. (2007), under Bayesian learning on \( d \), stock prices are no longer converging. Therefore, instead of considering infinitely lived agents, I consider here very long-lived but nevertheless finitely lived agents. This implies that agents live \( J \) periods, with \( J \) very high. More details on the choice of \( J \) are provided in the calibration part.
stock prices now write:

\[ P_t = \sum_{j=1}^{J-t} \beta^j D_t E_t[(D_{t+j} - D_t)^{1-\gamma}], \]  

(see proof in Appendix C). Hence, conditionally on \( d \), we have:

\[ E[(D_{t+j} - D_t)^{1-\gamma} | d, \sigma, I_t] = \exp[(1-\gamma)jm_t + 0.5(1-\gamma)^2 \sigma^2]. \]  

(15)

Therefore, integrating over the whole distribution of \( d \) yields:

\[ E[(D_{t+j} - D_t)^{1-\gamma} | \sigma, I_t] = \exp(0.5(1-\gamma)^2 \sigma^2) E[(1-\gamma)jm_t | I_t]. \]  

(16)

As \( d \) is believed to follow a normal distribution with parameters \( m_t \) and \( \sigma_t \) at the end of period \( t \), the last term is the expected value of a random variable following a log-normal distribution with parameters \((1-\gamma)jm_t\) and \((1-\gamma)j\sigma_t\). Therefore,

\[ E[(D_{t+j} - D_t)^{1-\gamma} | \sigma, I_t] = \exp(0.5(1-\gamma)^2 \sigma^2) \exp((1-\gamma)jm_t + 0.5(1-\gamma)^2 j^2 \sigma^2). \]  

(17)

Eventually, it yields the following pricing function:

\[ P_t = D_t \sum_{j=1}^{J-t} \beta^j \exp((1-\gamma)m_t j + 0.5(1-\gamma)^2 j^2 (\sigma^2 + \sigma)). \]  

(18)

It is immediate to see that under Bayesian learning, the price-dividend ratio is no longer a constant share of dividends, as it depends on the hyperparameters \( m_t \) and \( \sigma_t \) which are updated every period. Beliefs are the only time-dependent variables the price-dividend ratio depends on. Therefore, fluctuations in the price-dividend ratio over time –that is, non-fundamental fluctuations in prices– are driven by expectations. Comparative statics show that the price-dividend ratio in \( t \) increases in \( m_t \) if and only \( \gamma < 1 \), increases in \( \sigma_t \), in \( \sigma \), in \( \beta \) and increases in \( \gamma \) if \( m_t < 0 \).
(see Appendix D).

In addition, returns are no longer independently and identically distributed and they are negatively predicted by the lagged price-dividend ratio.

Indeed, if we define:

$$
\delta_t = \frac{P_t}{D_t} = \sum_{j=1}^{J-t} \beta^j \exp(0.5(1 - \gamma)^2 \sigma^2 + (1 - \gamma)m_tj + 0.5(1 - \gamma)^2 j^2 (\sigma_t^2 + \sigma)),
$$

we get:

$$
R_t = \frac{1 + \delta_t}{\delta_{t-1}} \exp(d + \sigma \varepsilon_t).
$$

Bayesian learning thus allows to fit qualitative features of the data. It implies that non-fundamental fluctuations in asset prices arise as soon as uncertainty on the estimated parameters is not null. Such fluctuations bring the stock price away from its fundamental value and generates additional volatility relative to that in dividends. To this respect, those fluctuations are inefficient. As they are driven by expectations, a natural policy to mitigate these fluctuations is to bring expectations closer to their rational expectations counterpart through communication policy and thus information disclosure. In the next section, I investigate the impact of information disclosure from a regulator (a financial market authority or a central bank for instance) who does not know the true parameter of the logged dividend growth process either and learns it through Bayesian updating as well.

## 4 Information disclosure and asset prices

I now model the regulator’s endogenous information on the true location parameter of the logged dividend growth. It learns it through the same inference process as economic agents, except that its degree of recency bias 0 ≤ α_R ≤ 1 is not restricted to be similar to that of the representative investor in the stock market. In the limiting case in which α_R = 1, the regulator does not suffer from recency bias.
Therefore, in that case, even if the regulator is not omniscient and does not know the true parameter, its estimate converges to the true one in the limit.

I investigate below the impact of the regulator’s information disclosure on the representative agent’s expectations and thus on the non-fundamental part of stocks price, depending on the regulator’s degree of recency bias.

4.1 The regulator’s endogenous information

When \(0 < \alpha_R < 1\), the regulator’s prior distribution for \(d\) is \(N \sim (m_{R,t}^p, \sigma_{R,t}^p)\) with

\[
m_{R,t}^p = \frac{y_t - 1\alpha_R \rho + y_{t-2}\alpha_R^2 \rho + \ldots + y_0 \alpha_R^t \rho}{(\alpha_R + \alpha_R^2 + \ldots + \alpha_R^t) * \rho},
\]

and

\[
\sigma_{R,t}^p = \frac{1}{(\alpha_R + \alpha_R^2 + \ldots + \alpha_R^t) * \rho}.
\]

Therefore, the regulator’s posterior distribution for \(d\) is \(N \sim (m_{R,t}, \sigma_{R,t})\) with

\[
m_{R,t} = \frac{y_t + y_{t-1}\alpha_R \rho + y_{t-2}\alpha_R^2 \rho + \ldots + y_0 \alpha_R^t \rho}{(1 + \alpha_R + \alpha_R^2 + \ldots + \alpha_R^t) * \rho},
\]

and

\[
\sigma_{R,t} = \frac{1}{(1 + \alpha_R + \alpha_R^2 + \ldots + \alpha_R^t) * \rho} = \frac{1 - \alpha_R}{(1 - \alpha_R^{t+1}) * \rho}.
\]

When \(\alpha_R = 1\), the regulator is not recency-biased and:

\[
m_{R,t} = \frac{y_t + y_{t-1}\rho + y_{t-2}\rho + \ldots + y_0 \rho}{(t + 1) * \rho},
\]

and

\[
\sigma_{R,t} = \frac{1}{(t + 1) * \rho}.
\]
4.2 Impact of information disclosure on expectations

In each period, the regulator displays its own information $N \sim (m_{R,t}, \sigma_{R,t})$ to the representative investor whose posterior distribution for $d$ is $N \sim (m_t, \sigma_t)$. Given that both beliefs are endogenously formed from the observation of past and current realizations of the logged dividend growth process and consequently are not independent, the representative investor processes the new information provided by the regulator by substituting it to its own beliefs when the regulator’s signal is more precise than his private signal and by ignoring it when it is less precise.

Let’s write $m_{\text{post},t}$ the location parameter of the representative’s agent posterior distribution of $d$ after the regulator displays its own signal, and $\sigma_{\text{post},t}$ the corresponding uncertainty parameter.

If $\sigma_t > \sigma_{R,t}$, that is, if $\alpha < \alpha_R$: $m_{\text{post},t} = m_{R,t}$ and $\sigma_{\text{post},t} = \sigma_t$.

If $\alpha > \alpha_R$, $m_{\text{post},t} = m_t$ and $\sigma_{\text{post},t} = \sigma_t$.

Following information disclosure, the new price-dividend ratio in each period $t$ writes:

$$\frac{P_t}{D_t} = \sum_{j=1}^{J-t} \beta^j \exp(0.5(1-\gamma)^2 \sigma^2 j + (1-\gamma)m_{\text{post},t} + 0.5(1-\gamma)^2 \sigma_{\text{post},t}^2 j^2).$$  \hspace{1cm} (26)

As it depends on $m_{\text{post},t}$ and on $\sigma_{\text{post},t}$, the price-dividend ratio is thus modified by information disclosure if and only if $\alpha < \alpha_R$.

In the extreme case in which the regulator knows the true parameter in the initial period, $m_{R,t} = 0$ and $\sigma_{R,t} = 0$ for $t \geq 1$. The price-dividend ratio following information disclosure writes:

$$\frac{P_t}{D_t} = \sum_{j=1}^{J-t} \beta^j \exp((1-\gamma)d_j + 0.5(1-\gamma)^2 \sigma^2 j^2).$$  \hspace{1cm} (27)
It thus reduces to the rational expectations price-dividend ratio for all $t$.

I now illustrate the mechanism of the model with a simulation exercise on the US stock market starting in the run-up to the most recent bust in the stock market, which followed the initial shock on the subprime loans market. Simulation results then allow to provide some elements of an answer to the maximal recency bias required in order to achieve a significant reduction in non-fundamental fluctuations. Quantitatively, it appears that only a very low degree of recency bias in the regulator’s learning process enables to achieve a significant decrease in volatility.

5 An illustration on the S&P 500 US stock market in the run-up to the subprime financial crisis and its aftermath

5.1 Simulation results

In order to assess quantitatively the impact of the regulator’s recency bias on the efficiency of information disclosure, I now calibrate the model on the US S&P 500 monthly data (Shiller’s dataset). Parameters of the dividend growth process are chosen so as to match those in the data over the period 1960-2014 (Table 1). $\beta$ is chosen based on prior literature whereas $\gamma$ and $\alpha$ are the two parameters on which some degree of freedom remains. Note however that $\gamma$ has to be inferior to 1 in order not to produce counterintuitive features that would contradict the data. Indeed, if $\gamma > 1$, then the wealth effect dominates the substitution effect, meaning that when the discounted expected present value of real dividends streams is expected to be higher, demand for stocks decreases whereas consumption increases, leading to a decrease in stock

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3 Monthly data on the US SP 500 stock market is retrieved from Shiller’s dataset available online.
prices. The values of $0 < \gamma < 1$ and $0 < \alpha < 1$ are chosen such that the qualitative evolution in the price-dividend ratio after the burst of the dot-com bubble is better replicated. The first justification to this is that the evolution of the price-dividend ratio since 2003 has been much less successfully accounted for in learning models that replicate well the previous historical evolution of the price-dividend ratio (see Adam et al. (2015) and Nakov and Nuno (2015)). The second justification is that it allows to assess to what extent information disclosure on the underlying fundamental process could have helped mitigating the extent of the burst in the US stock market during the recent subprime crisis.

$J$ is chosen so as to be high enough not to impact prices over the simulation period but such that simulations can still be computationally possible.

As for the initial prior distribution $d \sim N(m_1^p, \sigma_1^p)$, it arises endogenously from the out-sample prior information and consistently with the in-sample agents’ learning process: it is the parameter estimated from Bayesian inference with recency bias over previous out of sample realizations. Finally, the model is fed with the exact same dividends realizations as those observed in the data when deriving the model-implied price-dividend ratio. Over the recent period, the model generates

<table>
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<th>Parameter</th>
<th>Calibrated value</th>
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<tr>
<td>$d$</td>
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</tr>
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<td>$\sigma$</td>
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</tr>
<tr>
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<tr>
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</tr>
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<td>$J$</td>
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</tr>
</tbody>
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Table 1: Calibration

less volatility than what is observed in the data (Figure 1). Nevertheless, it repli-
cates the fact that the non-fundamental fluctuations in prices remain above the rational expectations implied ones in the run-up to the financial crisis and the bust in 2009, even though the bust in the data is steeper. In addition, the fact that the increase in the price-dividend ratio after the bust occurs a bit later in the model relative to the data suggests that a feedback learning model would not perform better as it would induce more persistence in the decrease in the price-dividend ratio. The model replicates the autocorrelation in the price-dividend ratio behavior over the period and predictability of the realized stock returns by the lagged price-dividend ratio (the inverse of the dividend yield) at several horizons (Table 2). P-values are displayed in brackets.

Therefore, if the model misses part of the volatility in the observed US price-dividend ratio over the recent period, it is nevertheless able to replicate lagged relationships between several variables, and to replicate the alternation of an episode where the price-dividend ratio is above the rational expectations value followed by an episode during which it quickly decreases below this value, before increas-

Figure 1: 2003-2014 Monthly price-dividend ratio
Table 2: Simulation results

<table>
<thead>
<tr>
<th>Autocorrelation in the P/D ratio</th>
<th>Subjective expectations model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation between returns and the lagged price-dividend ratio</td>
<td>-0.2538 (0.0027)</td>
<td>-0.2235 (0.0084)</td>
</tr>
<tr>
<td>Horizon: 6 months</td>
<td>-0.1796 (0.0425)</td>
<td>-0.1770 (0.0456)</td>
</tr>
<tr>
<td>Horizon: 16 months</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.2 The impact of information disclosure

I now assess quantitatively under which condition on the regulator’s beliefs formation process, information disclosure can be a relevant solution for mitigating volatility in asset prices. I consider only the cases for which $\alpha_R > \alpha$ (that is the regulator is less recency-biased than economic agents and its signal is thus more precise than that of agents). As explained above, when $\alpha_R < \alpha$, the representative agent’s private signal is more precise than that of the regulator (which is not independent), and it is optimal for the investor just to ignore it. The price-dividend ratio thus remains unchanged.

Figure 2 presents the evolution in the price-dividend ratio following information disclosure depending on the regulator’s degree of recency bias.

At first glance, it is striking that there are strong non-linearities in the impact of information disclosure on the volatility of the price-dividend ratio, depending on the regulator’s degree of recency bias. When the regulator is not recency-biased, information disclosure mitigates much more the non-fundamental volatility in prices than when it is even only slightly recency-biased.
Figure 2: The impact of information disclosure on the price-dividend ratio for various degrees of the regulator’s recency bias

I now assess under which condition on the degree of the regulator’s recency bias distinct objectives in terms of mitigating the volatility in the price-dividend ratio and bringing it closer to its rational expectations value could be achieved. The simulation results are displayed in Table 3.

Very small degrees of recency bias in the regulator learning process are required so as to achieve significant decrease in the price-dividend ratio volatility and in the average distance of the price-dividend ratio to its rational expectations value. Therefore, if information disclosure seems to be a relevant tool whenever the regulator is not recency-biased, as soon as it is itself recency-biased, this raises serious concerns on the ability of information disclosure to significantly mitigate non-fundamental fluctuations in asset prices. These results thus suggest that, in order to make information disclosure a useful tool in mitigating inefficient fluctuations in asset prices, more attention has to be paid to longer-span historical series of data, as recommended by Haldane (2009) or Reinhart and Rogoff (2009).
### Table 3: Minimal degree of the regulator’s recency bias required in order to achieve distinct objectives

<table>
<thead>
<tr>
<th>Objective</th>
<th>Minimal $\alpha_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance of P/D: &lt;1% of the RE value of P/D</td>
<td>0.99</td>
</tr>
<tr>
<td>Variance of P/D: &lt;5% of the RE value of P/D</td>
<td>0.97</td>
</tr>
<tr>
<td>Variance of P/D: &lt;10% of the RE value of P/D</td>
<td>0.96</td>
</tr>
<tr>
<td>Mean of the distance of P/D to its RE value: &lt;1% of the RE value</td>
<td>1</td>
</tr>
<tr>
<td>Mean of the distance of P/D to its RE value: &lt;5% of the RE value</td>
<td>0.98</td>
</tr>
<tr>
<td>Mean of the distance of P/D to its RE value: &lt;10% of the RE value</td>
<td>0.95</td>
</tr>
</tbody>
</table>

#### 6 Conclusion

In a simple standard consumption-based asset pricing model in which agents learn the location parameter of the dividend growth process through Bayesian inference and which provides microfoundations to investors’ decision without implying any restrictive assumption on agents’ knowledge that their estimate is uncertain and on agents’ knowledge of the pricing function, it is possible to derive a closed-form solution for stock price. This makes obvious how it depends on investors’ expectations and generates fluctuations in the price-dividend ratio and thus the potential for non-fundamental bubbles. The specificity of the model is that the extent and the persistence of these fluctuations over time are due to the representative investor’s recency bias, relying on empirical evidence.

The model proves able to replicate some features of the US stock market in the run-up to the subprime crisis, its outbreak and its aftermath: the price-dividend ratio is not constant over time and evolves according to surprise effects –thus displaying a steep decrease in the beginning of 2009–, it is strongly autocorrelated, and returns are predictable by the lagged price-dividend ratio at several horizons. As expectations-driven booms and busts arise, this paves the way for communi-
cation policy from the regulator. Nevertheless, information disclosure on the true model parameter significantly mitigates expectations-driven fluctuations only when the regulator’s recency bias is small enough relative to that of agents. Those results suggest that communication policy from the regulator should not only target forward-looking expectations but also backward-looking ones. To this aim, it matters that not only recent data is processed but also earlier historical one, what would allow to better identify unusual behavior in asset prices and other macro-financial variables relative to their historical behavior.

References


A Proof of Proposition 1

The first Euler equation (with respect to quantity of stocks $S_t$) writes:

$$D_t^{-\gamma} = \beta E_t[D_{t+1}(\frac{P_{t+1} + D_{t+1}}{P_t})]. \quad (28)$$

Isolating $P_t$ on the left hand side yields:

$$P_t = \beta E_t[(\frac{D_{t+1}}{D_t})^{-\gamma}(P_{t+1} + D_{t+1})]. \quad (29)$$

Substituting $P_{t+1}$ by its expression in the iterated forward version of the previous equation yields:

$$P_t = \beta E_t[(\frac{D_{t+1}}{D_t})^{-\gamma}(\beta E_{t+1}[\frac{D_{t+2}}{D_{t+1}}]^{-\gamma}(P_{t+2} + D_{t+2})) + (\frac{D_{t+1}}{D_t})^{-\gamma}D_{t+1}] \quad (30)$$

Applying the law of iterated expectations ($E_t[E_{t+1}(X)] = E_t[X]$) and iterating forward again yields:

$$P_t = E_t[\beta(\frac{D_{t+1}}{D_t})^{-\gamma}P_{t+1}] + E_t[\beta(\frac{D_{t+1}}{D_t})^{-\gamma}D_{t+1} + \beta(\frac{D_{t+2}}{D_t})^{-\gamma}D_{t+2} + \ldots + \beta^J((\frac{D_{t+J}}{D_t})^{-\gamma}D_{t+J})]. \quad (31)$$

Imposing the following transversality condition $\lim_{J \to \infty} E_t[\beta(\frac{D_{t+J}}{D_t})^{-\gamma}P_{t+1}] = 0$ in the previous equation, one gets:

$$P_t = E_t \sum_{j=1}^{\infty} \frac{\beta^j D_{t+1}^{\gamma}}{D_t^\gamma} D_{t+j}. \quad (32)$$
Thus,
\[
E_t[(D_{t+j} \frac{D_{t+j-1}}{D_{t+j-2}} \ldots \frac{D_{t+1}}{D_t})^{1-\gamma}] = E_t[E_{t+1}[(D_{t+j} \frac{D_{t+j-1}}{D_{t+j-2}} \ldots E_{t+j}[\frac{D_{t+j}}{D_{t+j-1}}])^{1-\gamma}]]
\]
(law of iterated expectations). Hence,
\[
E_t[(D_{t+j} \frac{D_{t+j-1}}{D_{t+j-2}})^{1-\gamma}] = \exp(d(1-\gamma) + \frac{(1-\gamma)^2 \sigma_n^2}{2}) = \theta.
\]
Therefore,
\[
E_t[(D_{t+j} \frac{D_{t}}{D_t})^{1-\gamma}] = \theta^j,
\]
and
\[
P_t = D_t \lim_{J \to \infty} \sum_{j=1}^{J} \beta^j \theta^j.
\]
It is the sum of the J first terms of a geometric sequence with common ratio \(\beta \theta\) and first term \(\beta \theta\). Therefore, if and only if \(\beta \theta < 1\),
\[
P_t = \frac{\beta \theta}{1 - \beta \theta}.
\]

\[\text{B} \quad \text{Inconsistent features of the rational expectations model}\]

\[\text{B.1 Constant versus volatile price-dividend ratio}\]

The following chart displays the US S&P 500 monthly price-dividend ratio over the recent period and the rational expectations model implied one, which is constant (parameters values used here are the same as those presented in the simulation exercise in Section 5.)
B.2 Iid returns versus autocorrelated returns

The following chart presents realized monthly returns on US S&P 500 stocks. They are obviously non iid, whereas the rational expectations model predict them to be.
C Proof of Proposition 2

Under learning, in each period, investors maximize the discounted present value sum of their present and expected future consumption conditionally on their information in period \( t \) and thus on their Bayesian estimates in period \( t \).

In penultimate period \( J-1 \), the first order condition of the agent’s maximization program writes:

\[
C_{J-1}^{\gamma} = \beta E_{J-1}[C_{J-1}^{\gamma}(\frac{D_J}{P_{J-1}})].
\]

Relying on the law of iterated expectations and forward iteration as in proof of Proposition 1, it yields (under the non-bequest assumption):

\[
P_{J-1} = \beta D_{J-1}E_{J-1}[\frac{D_J}{D_{J-1}}]^{1-\gamma}.
\]

(38)

Similarly, in period \( J-2 \), stock price writes:

\[
P_{J-2} = \beta D_{J-2}E_{J-2}[\frac{D_J}{D_{J-2}}]^{1-\gamma} + \beta E_{J-2}D_{J-2}[\frac{D_J}{D_{J-1}}]^{1-\gamma}(\frac{D_{J-1}}{D_{J-2}})^{1-\gamma}.
\]

(39)

Eventually, in each period \( t \):

\[
P_t = \sum_{j=1}^{J-t} \beta^j D_tE_t[\frac{D_{t+j}}{D_t}]^{1-\gamma}.
\]

(40)

D Comparative statics

As \( P_t = D_t \sum_{j=1}^{J-t} \beta^j \exp((1-\gamma)m_{t,j} + 0.5(1-\gamma)j^2(\sigma_t^2 + \sigma^2)) \), the sign of its derivative with respect to any variable is the same as the sign of the derivative of \( \beta^j \exp((1-\gamma)m_{t,j} + 0.5(1-\gamma)j^2(\sigma_t^2 + \sigma^2)) \) with respect to that variable. I now write \( \beta^j \exp((1-\gamma)m_{t,j} + 0.5(1-\gamma)j^2(\sigma_t^2 + \sigma^2)) = x_t \).

\[
\frac{\partial x_t}{\partial \beta} = j \beta^{j-1} \exp((1-\gamma)m_{t,j} + 0.5(1-\gamma)j^2(\sigma_t^2 + \sigma^2)) > 0.
\]
When the discount rate is higher—that is preference for present consumption is lower—the demand for stock prices increases and thus the equilibrium stock price increases.

\[
\frac{\partial x_t}{\partial \gamma} = \beta j \exp((1 - \gamma)m_t j + 0.5(1 - \gamma)^2 j^2(\sigma_t^2 + \sigma^2))(-m_t - (1 - \gamma)(\sigma^2 + \sigma_t^2)j).
\]

\[
\frac{\partial x_t}{\partial \gamma} > 0 \iff m_t < (1 - \gamma)(\sigma^2 + \sigma_t^2)j.
\]

The sign of the derivative of the stock price with respect to the relative risk aversion coefficient depends on the hyperparameters \(m_t\) and \(\sigma_t\) and thus on the data realizations. If \(\gamma < 1\), the substitution effect dominates the wealth effect, and thus present consumption decreases when the expected payoff on stocks increases. In this case, \((1 - \gamma)(\sigma^2 + \sigma_t^2)j > 0\). Therefore, when relative risk aversion increases, the demand for stocks increases if and only if the expected mean of the logged growth rate of the payoff on stocks is small enough relative to the uncertainty on this parameter and the variance of the process.

\[
\frac{\partial x_t}{\partial \sigma} = \beta j^2 \exp((1 - \gamma)m_t j + 0.5(1 - \gamma)^2 j^2(\sigma_t^2 + \sigma^2))(1 - \gamma)^2 \sigma > 0.
\]

The demand for stocks increases when the (known) variance of the logged growth rate of the payoff on stocks increases.

\[
\frac{\partial x_t}{\partial \sigma_t} = \beta j^2 \exp((1 - \gamma)m_t j + 0.5(1 - \gamma)^2 j^2(\sigma_t^2 + \sigma^2))(1 - \gamma)^2 \sigma_t > 0.
\]

Similarly, the demand for stocks increases when the uncertainty on the expected mean of the logged growth rate of the payoff on stocks increases.

\[
\frac{\partial x_t}{\partial m_t} = \beta j \exp((1 - \gamma)m_t j + 0.5(1 - \gamma)^2 j^2(\sigma_t^2 + \sigma^2))(1 - \gamma).
\]
\[ \frac{\partial x_t}{\partial m_t} > 0 \Leftrightarrow \gamma < 1. \] When the substitution effect dominates the wealth effect, demand for stocks increases when the expected mean of the logged growth rate of the payoff on stocks increases.